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THE $\{-2, -1\}$-SELFDUAL AND DECOMPOSABLE TOURNAMENTS

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Abstract

We only consider finite tournaments. The dual of a tournament is obtained by reversing all the arcs. A tournament is *selfdual* if it is isomorphic to its dual. Given a tournament T, a subset X of $V(T)$ is a *module* of T if each vertex outside X dominates all the elements of X or is dominated by all the elements of X. A tournament T is *decomposable* if it admits a module X such that $1 < |X| < |V(T)|$.

We characterize the decomposable tournaments whose subtournaments obtained by removing one or two vertices are selfdual. We deduce the following result. Let T be a non decomposable tournament. If the subtournaments of T obtained by removing two or three vertices are selfdual, then the subtournaments of T obtained by removing a single vertex are not decomposable. Lastly, we provide two applications to tournaments reconstruction.

Keywords: tournament, decomposable, selfdual.

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