Discussiones Mathematicae
Graph Theory 38 (2018) 743–789
doi:10.7151/dmgt.2059

THE $\{-2,-1\}$-SELFDUAL AND DECOMPOSABLE TOURNAMENTS

Youssef Boudabbous[1]

Department of Mathematics
King Saud University
P.O. Box 2455, Riyadh 11451, Saudi Arabia

e-mail: yboudabbous@ksu.edu.sa

and

Pierre Ille

Aix Marseille University
CNRS, Centrale Marseille, I2M
Marseille, France

e-mail: pierre.ille@univ-amu.fr

Abstract

We only consider finite tournaments. The dual of a tournament is obtained by reversing all the arcs. A tournament is *selfdual* if it is isomorphic to its dual. Given a tournament T, a subset X of $V(T)$ is a *module* of T if each vertex outside X dominates all the elements of X or is dominated by all the elements of X. A tournament T is *decomposable* if it admits a module X such that $1 < |X| < |V(T)|$.

We characterize the decomposable tournaments whose subtournaments obtained by removing one or two vertices are selfdual. We deduce the following result. Let T be a non decomposable tournament. If the subtournaments of T obtained by removing two or three vertices are selfdual, then the subtournaments of T obtained by removing a single vertex are not decomposable. Lastly, we provide two applications to tournaments reconstruction.

Keywords: tournament, decomposable, selfdual.

2010 Mathematics Subject Classification: 05C20, 05C75, 05C60.

[1]The first author would like to extend his sincere appreciation to the Deanship of Scientific Research at King Saud University for its funding of this research through the Research Group Project No RGP-056.

References

[1] M. Achour, Y. Boudabbous and A. Boussaïri, *The $\{-3\}$-reconstruction and the $\{-3\}$-self duality of tournaments*, Ars Combin. **122** (2015) 355–377.

[2] M. Basso-Gerbelli and P. Ille, *La reconstruction des relations définies par interdits*, C. R. Acad. Sci. Paris, Sér. I Math. **316** (1993) 1229–1234.

[3] H. Belkhechine, I. Boudabbous and J. Dammak, *Morphologie des tournois* (-1)-*critiques*, C. R. Acad. Sci. Paris, Sér. I Math. **345** (2007) 663–666.
doi:10.1016/j.crma.2007.11.006

[4] A. Bondy and R.L. Hemminger, *Graph reconstruction, a survey*, J. Graph Theory **1** (1977) 227–268.
doi:10.1002/jgt.3190010306

[5] H. Bouchaala, *Sur la répartition des diamants dans un tournoi*, C. R. Acad. Sci. Paris, Sér. I Math. **338** (2004) 109–112.
doi:10.1016/j.crma.2003.11.018

[6] H. Bouchaala and Y. Boudabbous, *La $\{-k\}$-autodualité des sommes lexicographiques finies de tournois suivant un* 3*-cycle ou un tournoi critique*, Ars Combin. **81** (2006) 33–64.

[7] Y. Boudabbous, J. Dammak and P. Ille, *Indecomposability and duality of tournaments*, Discrete Math. **223** (2000) 55–82.
doi:10.1016/S0012-365X(00)00040-6

[8] Y. Boudabbous and A. Boussaïri, *Reconstruction des tournois et dualité*, C. R. Acad. Sci. Paris, Sér. I Math. **320** (1995) 397–400.

[9] Y. Boudabbous and P. Ille, *Indecomposability graph and critical vertices of an indecomposable graph*, Discrete Math. **309** (2009) 2839–2846.
doi:10.1016/j.disc.2008.07.015

[10] Y. Boudabbous and P. Ille, *Cut-primitive directed graphs versus clan-primitive directed graphs*, Adv. Pure Appl. Math. **1** (2010) 223–231.
doi:10.1515/apam.2010.013

[11] A. Boussaïri, Décomposabilité, dualité et groupes finis en théorie des relations (Ph.D. Thesis, Université Claude Bernard, Lyon I, 1995).

[12] A. Boussaïri, P. Ille, G. Lopez and S. Thomassé, *The C_3-structure of the tournaments*, Discrete Math. **277** (2004) 29–43.
doi:10.1016/S0012-365X(03)00244-9

[13] A. Cournier and M. Habib, *A new linear algorithm for modular decomposition*, in: Trees in Algebra and Programming, S. Tison (Ed(s)), (Springer, 1994) 68–84.
doi:10.1007/BFb0017474

[14] A. Ehrenfeucht, T. Harju and G. Rozenberg, The Theory of 2-Structures, A Framework for Decomposition and Transformation of Graphs (World Scientific, 1999).
doi:10.1142/4197

[15] W.J.R. Eplett, *Self-converse tournaments*, Canad. Math. Bull. **22** (1979) 23–27. doi:10.4153/CMB-1979-004-6

[16] R. Fraïssé, Theory of Relations, Revised Edition (North-Holland, 2000).

[17] T. Gallai, *Transitiv orientierbare Graphen*, Acta Math. Acad. Sci. Hungar. **18** (1967) 25–66.
doi:10.1007/BF02020961

[18] F. Harary and E. Palmer, *On the problem of reconstructing a tournament from subtournaments*, Monatsh. Math. **71** (1967) 14–23.
doi:10.1007/BF01299955

[19] P. Ille, *La reconstruction des relations binaires*, C. R. Acad. Sci. Paris, Sér. I Math. **306** (1988) 635–638.

[20] P. Ille, *Recognition problem in reconstruction for decomposable relations*, in: Finite and Infinite Combinatorics in Sets and Logic, B. Sands, N. Sauer and R. Woodrow (Ed(s)), (Kluwer Academic Publishers, 1993) 189–198.
doi:10.1007/978-94-011-2080-7_13

[21] W.M. Kantor, *Automorphism groups of designs*, Math. Z. **109** (1969) 246–252.
doi:10.1007/BF01111409

[22] W.M. Kantor, *Automorphism groups of designs*, Math. Z. **109** (1969) 246–252.
doi:10.1007/BF01111409

[23] G. Lopez, *Deux résultats concernant la détermination d'une relation par les types d'isomorphie de ses restrictions*, C. R. Acad. Sci. Paris, Sér. A-B **274** (1972) 1525–1528.

[24] G. Lopez, *L'indéformabilité des relations et multirelations binaires*, Z. Math. Logik Grundlag. Math. **24** (1978) 303–317.
doi:10.1002/malq.19780241905

[25] G. Lopez and C. Rauzy, *Reconstruction of binary relations from their restrictions of cardinality* $2, 3, 4$ *and* $(n-1)$, II, Z. Math. Logik Grundlag. Math. **38** (1992) 157–168.
doi:10.1002/malq.19920380111

[26] F. Maffray and M. Preissmann, *A translation of Tibor Gallai's paper: Transitiv orientierbare Graphen*, in: Perfect Graphs, J.L. Ramirez-Alfonsin and B.A. Reed (Ed(s)), (Wiley, 2001) 25–66.

[27] J.W. Moon, *Tournaments whose subtournaments are irreducible or transitive*, Canad. Math. Bull. **22** (1979) 75–79.
doi:10.4153/CMB-1979-010-7

[28] M. Pouzet, *Application d'une propriété combinatoire des parties d'un ensemble aux groupes et aux relations*, Math. Z. **150** (1976) 117–134.
doi:10.1007/BF01215230

[29] K.B. Reid and C. Thomassen, *Strongly self-complementarity and hereditarily isomorphic tournaments*, Monatsh. Math. **81** (1976) 291–304.
doi:10.1007/BF01387756

[30] M.Y. Sayar, *Partially critical indecomposable tournaments and partially critical supports*, Contrib. Discrete Math. **6** (2011) 52–76.

[31] J.H. Schmerl and W.T. Trotter, *Critically indecomposable partially ordered sets, graphs, tournaments and other binary relational structures*, Discrete Math. **113** (1993) 191–205.
doi:10.1016/0012-365X(93)90516-V

[32] J. Spinrad, P_4*-trees and substitution decomposition*, Discrete Appl. Math. **39** (1992) 263–291.
doi:10.1016/0166-218X(92)90180-I

[33] P.K. Stockmeyer, *The falsity of the reconstruction conjecture for tournaments*, J. Graph Theory **1** (1977) 19–25.
doi:10.1002/jgt.3190010108

[34] S.M. Ulam, A Collection of Mathematical Problems (Intersciences Publishers, 1960).

Received 3 August 2016
Revised 6 February 2017
Accepted 6 February 2017