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TOTAL COLORINGS OF EMBEDDED GRAPHS WITH NO 3-CYCLES ADJACENT TO 4-CYCLES

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Abstract

A *total-k-coloring* of a graph G is a coloring of $V \cup E$ using k colors such that no two adjacent or incident elements receive the same color. The *total chromatic number* $\chi''(G)$ of G is the smallest integer k such that G has a total-k-coloring. Let G be a graph embedded in a surface of Euler characteristic $\varepsilon \geq 0$. If G contains no 3-cycles adjacent to 4-cycles, that is, no 3-cycle has a common edge with a 4-cycle, then $\chi''(G) \leq \max\{8, \Delta + 1\}$.

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References

[1] M. Behzad, Graphs and Their Chromatic Numbers (Ph.D. Thesis, Michigan State University, 1965).

[2] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications (Macmillan Press Ltd., London, 1976).

[3] O.V. Borodin, *On the total coloring of planar graphs*, J. Reine Angew. Math. **394** (1989) 180–185.

[4] O.V. Borodin, *Coupled colourings of graphs on a plane*, Metody Diskret. Anal. **45** (1987) 21–27, in Russian.

[5] O.V. Borodin, A.V. Kostochka and D.R. Woodall, *List edge and list total colourings of multigraphs*, J. Combin. Theory Ser. B **71** (1997) 184–204. doi:10.1006/jctb.1997.1780

[6] O.V. Borodin A.V. Kostochka and D.R. Woodall, *Total colorings of planar graphs with large maximum degree*, J. Graph Theory **26** (1997) 53–59. doi:10.1002/(SICI)1097-0118(199709)26:1⟨53::AID-JGT6⟩3.0.CO;2-G

[7] G.J. Chang, J. Hou and N. Roussel, *Local condition for planar graphs of maximum degree* 7 *to be* 8*-totally colorable*, Discrete Appl. Math. **159** (2011) 760–768. doi:10.1016/j.dam.2011.01.001

[8] D. Du, L. Shen and Y. Wang, *Planar graphs with maximum degree* 8 *and without adjacent triangles are* 9*-totally-colorable*, Discrete Appl. Math. **157** (2009) 2778–2784. doi:10.1016/j.dam.2009.02.011

[9] T.R. Jensen and B. Toft, Graph Coloring Problems (Wiley Interscience, 1995).

[10] Ł. Kowalik, J.-S. Sereni and R. Škrekovski, *Total-colorings of plane graphs with maximum degree nine*, SIAM J. Discrete Math. **22** (2008) 1462–1479. doi:10.1137/070688389

[11] L. Shen and Y.Q. Wang, *Total colorings of planar graphs with maximum degree at least* 8, Sci. China Ser A: Math. **52** (2009) 1733–1742. doi:10.1007/s11425-008-0155-3

[12] L. Shen and Y. Wang, *On the* 7 *total colorability of planar graphs with maximum degree* 6 *and without* 4*-cycles*, Graphs Combin. **25** (2009) 401–407. doi:10.1007/s00373-009-0843-y

[13] A.V. Kostochka, *The total coloring of a multigraph with maximal degree* 4, Discrete Math. **17** (1977) 161–163. doi:10.1016/0012-365X(77)90146-7

[14] A.V. Kostochka, *An analogue of Shannon's estimate for complete colorings*, Metody Diskret. Anal. **30** (1977) 13–22, in Russian.

[15] A.V. Kostochka, *The total chromatic number of any multigraph with maximum degree five is at most seven*, Discrete Math. **162** (1996) 199–214. doi:10.1016/0012-365X(95)00286-6

[16] B. Liu, J.F. Hou, J.L. Wu and G.Z. Liu, *Total colorings and list total colorings of planar graphs without intersecting* 4*-cycles*, Discrete Math. **309** (2009) 6035–6043. doi:10.1016/j.disc.2009.05.006

[17] D.P. Sanders and Y. Zhao, *On total* 9*-coloring planar graphs of maximum degree seven*, J. Graph Theory **31** (1999) 67–73.
doi:10.1002/(SICI)1097-0118(199905)31:1⟨67::AID-JGT6⟩3.0.CO;2-C

[18] V.G. Vizing, *Some unsolved problems in graph theory*, Uspekhi Mat. Nauk **23** (1968) 117–134, in Russian.

[19] B. Wang and J.-L. Wu, *Total coloring of planar graphs with maximum degree seven*, Inform. Process. Lett. **111** (2011) 1019–1021.
doi:10.1016/j.ipl.2011.07.012

[20] P. Wang and J.-L. Wu, *A note on total colorings of planar graphs without* 4*-cycles*, Discuss. Math. Graph Theory **24** (2004) 125–135.
doi:10.7151/dmgt.1219

[21] H.J. Wang, L.D. Wu, W.L. Wu, P.M. Pardalos and J.L. Wu, *Minimum total coloring of planar graph*, J. Global Optim. **60** (2014) 777–791.
doi:10.1007/s10898-013-0138-y

[22] H.J. Wang, B. Liu, J.L. Wu and G.Z. Liu, *Total coloring of embedded graphs with maximum degree at least seven*, Theoret. Comput. Sci. **518** (2014) 1–9.
doi:10.1016/j.tcs.2013.04.030

[23] H.J. Wang, B. Liu, J.L. Wu and B. Wang, *Total coloring of graphs embedded in surfaces of nonnegative Euler characteristic*, Sci. China Math. **57** (2014) 211–220.
doi:10.1007/s11425-013-4576-2

[24] J.L. Wu and P. Wang, *List-edge and list-total colorings of graphs embedded on hyperbolic surfaces*, Discrete Math. **308** (2008) 6210–6215.
doi:10.1016/j.disc.2007.11.044

[25] W.F. Wang, *Total chromatic number of planar graphs with maximum degree ten*, J. Graph Theory **54** (2007) 91–102.
doi:10.1002/jgt.20195

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