

TOTAL COLORINGS OF EMBEDDED GRAPHS WITH NO 3-CYCLES ADJACENT TO 4-CYCLES

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Abstract

A *total- k -coloring* of a graph G is a coloring of $V \cup E$ using k colors such that no two adjacent or incident elements receive the same color. The *total chromatic number* $\chi''(G)$ of G is the smallest integer k such that G has a total- k -coloring. Let G be a graph embedded in a surface of Euler characteristic $\varepsilon \geq 0$. If G contains no 3-cycles adjacent to 4-cycles, that is, no 3-cycle has a common edge with a 4-cycle, then $\chi''(G) \leq \max\{8, \Delta + 1\}$.

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REFERENCES

- [1] M. Behzad, *Graphs and Their Chromatic Numbers* (Ph.D. Thesis, Michigan State University, 1965).

- [2] J.A. Bondy and U.S.R. Murty, *Graph Theory with Applications* (Macmillan Press Ltd., London, 1976).
- [3] O.V. Borodin, *On the total coloring of planar graphs*, J. Reine Angew. Math. **394** (1989) 180–185.
- [4] O.V. Borodin, *Coupled colourings of graphs on a plane*, Metody Diskret. Anal. **45** (1987) 21–27, in Russian.
- [5] O.V. Borodin, A.V. Kostochka and D.R. Woodall, *List edge and list total colourings of multigraphs*, J. Combin. Theory Ser. B **71** (1997) 184–204.
doi:10.1006/jctb.1997.1780
- [6] O.V. Borodin A.V. Kostochka and D.R. Woodall, *Total colorings of planar graphs with large maximum degree*, J. Graph Theory **26** (1997) 53–59.
doi:10.1002/(SICI)1097-0118(199709)26:1<53::AID-JGT6>3.0.CO;2-G
- [7] G.J. Chang, J. Hou and N. Roussel, *Local condition for planar graphs of maximum degree 7 to be 8-totally colorable*, Discrete Appl. Math. **159** (2011) 760–768.
doi:10.1016/j.dam.2011.01.001
- [8] D. Du, L. Shen and Y. Wang, *Planar graphs with maximum degree 8 and without adjacent triangles are 9-totally-colorable*, Discrete Appl. Math. **157** (2009) 2778–2784.
doi:10.1016/j.dam.2009.02.011
- [9] T.R. Jensen and B. Toft, *Graph Coloring Problems* (Wiley Interscience, 1995).
- [10] L. Kowalik, J.-S. Sereni and R. Škrekovski, *Total-colorings of plane graphs with maximum degree nine*, SIAM J. Discrete Math. **22** (2008) 1462–1479.
doi:10.1137/070688389
- [11] L. Shen and Y.Q. Wang, *Total colorings of planar graphs with maximum degree at least 8*, Sci. China Ser A: Math. **52** (2009) 1733–1742.
doi:10.1007/s11425-008-0155-3
- [12] L. Shen and Y. Wang, *On the 7 total colorability of planar graphs with maximum degree 6 and without 4-cycles*, Graphs Combin. **25** (2009) 401–407.
doi:10.1007/s00373-009-0843-y
- [13] A.V. Kostochka, *The total coloring of a multigraph with maximal degree 4*, Discrete Math. **17** (1977) 161–163.
doi:10.1016/0012-365X(77)90146-7
- [14] A.V. Kostochka, *An analogue of Shannon’s estimate for complete colorings*, Metody Diskret. Anal. **30** (1977) 13–22, in Russian.
- [15] A.V. Kostochka, *The total chromatic number of any multigraph with maximum degree five is at most seven*, Discrete Math. **162** (1996) 199–214.
doi:10.1016/0012-365X(95)00286-6
- [16] B. Liu, J.F. Hou, J.L. Wu and G.Z. Liu, *Total colorings and list total colorings of planar graphs without intersecting 4-cycles*, Discrete Math. **309** (2009) 6035–6043.
doi:10.1016/j.disc.2009.05.006

- [17] D.P. Sanders and Y. Zhao, *On total 9-coloring planar graphs of maximum degree seven*, J. Graph Theory **31** (1999) 67–73.
doi:10.1002/(SICI)1097-0118(199905)31:1<67::AID-JGT6>3.0.CO;2-C
- [18] V.G. Vizing, *Some unsolved problems in graph theory*, Uspekhi Mat. Nauk **23** (1968) 117–134, in Russian.
- [19] B. Wang and J.-L. Wu, *Total coloring of planar graphs with maximum degree seven*, Inform. Process. Lett. **111** (2011) 1019–1021.
doi:10.1016/j.ipl.2011.07.012
- [20] P. Wang and J.-L. Wu, *A note on total colorings of planar graphs without 4-cycles*, Discuss. Math. Graph Theory **24** (2004) 125–135.
doi:10.7151/dmgt.1219
- [21] H.J. Wang, L.D. Wu, W.L. Wu, P.M. Pardalos and J.L. Wu, *Minimum total coloring of planar graph*, J. Global Optim. **60** (2014) 777–791.
doi:10.1007/s10898-013-0138-y
- [22] H.J. Wang, B. Liu, J.L. Wu and G.Z. Liu, *Total coloring of embedded graphs with maximum degree at least seven*, Theoret. Comput. Sci. **518** (2014) 1–9.
doi:10.1016/j.tcs.2013.04.030
- [23] H.J. Wang, B. Liu, J.L. Wu and B. Wang, *Total coloring of graphs embedded in surfaces of nonnegative Euler characteristic*, Sci. China Math. **57** (2014) 211–220.
doi:10.1007/s11425-013-4576-2
- [24] J.L. Wu and P. Wang, *List-edge and list-total colorings of graphs embedded on hyperbolic surfaces*, Discrete Math. **308** (2008) 6210–6215.
doi:10.1016/j.disc.2007.11.044
- [25] W.F. Wang, *Total chromatic number of planar graphs with maximum degree ten*, J. Graph Theory **54** (2007) 91–102.
doi:10.1002/jgt.20195

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