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RAINBOW TOTAL-COLORING OF COMPLEMENTARY GRAPHS AND ERDŐS-GALLAI TYPE PROBLEM FOR THE RAINBOW TOTAL-CONNECTION NUMBER

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Abstract

A total-colored graph G is rainbow total-connected if any two vertices of G are connected by a path whose edges and internal vertices have distinct colors. The rainbow total-connection number, denoted by $rtc(G)$, of a graph G is the minimum number of colors needed to make G rainbow total-connected. In this paper, we prove that $rtc(G)$ can be bounded by a constant 7 if the following three cases are excluded: $diam(\overline{G}) = 2$, $diam(\overline{G}) = 3$, $\overline{G}$ contains exactly two connected components and one of them is a trivial graph. An example is given to show that this bound is best possible. We also study Erdős-Gallai type problem for the rainbow total-connection number, and compute the lower bounds and precise values for the function $f(n, k)$,

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where $f(n,k)$ is the minimum value satisfying the following property: if $|E(G)| \geq f(n,k)$, then $rtc(G) \leq k$.

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References

[1] J.A. Bondy and U.S.R. Murty, Graph Theory, GTM 244 (Springer, Berlin, 2008).

[2] S. Chakraborty, E. Fischer, A. Matsliah and R. Yuster, *Hardness and algorithms for rainbow connection*, J. Comb. Optim. **21** (2011) 330–347. doi:10.1007/s10878-009-9250-9

[3] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *Rainbow connection in graphs*, Math. Bohem. **133** (2008) 85–98.

[4] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *The rainbow connectivity of a graph*, Networks **54(2)** (2009) 75–81. doi:10.1002/net.20296

[5] L. Chen, B. Huo and Y. Ma, *Hardness results for total rainbow connection of graphs*, Discuss. Math. Graph Theory **36** (2016) 355–362. doi:10.7151/dmgt.1856

[6] L. Chen, X. Li and Y. Shi, *The complexity of determining the rainbow vertex-connection of a graph*, Theoret. Comput. Sci. **412** (2011) 4531–4535. doi:10.1016/j.tcs.2011.04.032

[7] X. Huang, H. Li, X. Li and Y. Sun, *Oriented diameter and rainbow connection number of a graph*, Discrete Math. Theor. Comput. Sci. **16(3)** (2014) 51–60.

[8] X. Huang, X. Li, Y. Shi, J. Yue and Y. Zhao, *Rainbow connections for outerplanar graphs with diameter* 2 *and* 3, Appl. Math. Comput. **242** (2014) 277–280. doi:10.1016/j.amc.2014.05.066

[9] A. Kemnitz and I. Schiermeyer, *Graphs with rainbow connection number two*, Discuss. Math. Graph Theory **31** (2011) 313–320. doi:10.7151/dmgt.1547

[10] M. Krivelevich and R. Yuster, *The rainbow connection of a graph is* (*at most*) *reciprocal to its minimum degree*, J. Graph Theory **63** (2010) 185–191. doi:10.1002/jgt.20418

[11] H. Li, X. Li, Y. Sun and Y. Zhao, *Note on minimally d-rainbow connected graphs*, Graphs Combin. **30** (2014) 949–955. doi:/10.1007/s00373-013-1309-9

[12] X. Li, Y. Shi and Y. Sun, *Rainbow connections of graphs: A survey*, Graphs Combin. **29** (2013) 1–38. doi:10.1007/s00373-012-1243-2

[13] X. Li and Y. Sun, *Rainbow connection numbers of line graphs*, Ars Combin. **100** (2011) 449–463.

[14] X. Li and Y. Sun, *Rainbow connection numbers of complementary graphs*, Util. Math. **86** (2011) 23–31.

[15] X. Li and Y. Sun, *Upper bounds for the rainbow connection numbers of line graphs*, Graphs Combin. **28** (2012) 251–263.
doi:10.1007/s00373-011-1034-1

[16] X. Li and Y. Sun, Rainbow Connections of Graphs, SpringerBriefs in Math. (Springer, New York, 2012).
doi:10.1007/978-1-4614-3119-0

[17] X. Li and Y. Sun, *On the strong rainbow connection of a graph*, Bull. Malays. Math. Sci. Soc. **36** (2013) 299–311.

[18] H. Liu, Â. Mestre and T. Sousa, *Total rainbow k-connection in graphs*, Discrete Appl. Math. **174** (2014) 92–101.
doi:10.1016/j.dam.2014.04.012

[19] Y. Ma, *Total rainbow connection number and complementary graph*, Results Math. **70** (2016) 173–182.
doi:10.1007/s00025-015-0469-8

[20] Y. Sun, *On two variants of rainbow connection*, WSEAS Trans. Math. **12** (2013) 266–276.

[21] Y. Sun, *On rainbow total-coloring of a graph*, Discrete Appl. Math. **194** (2015) 171–177.
doi:10.1016/j.dam.2015.05.012

[22] Y. Sun, *On the total rainbow connection of a graph*, Acta Math. Appl. Sin. Engl. Ser., accepted.

[23] Y. Sun, Z. Jin and F. Li, *On total rainbow k-connected graphs*, Appl. Math. Comput. **311** (2017) 223–227.
doi:10.1016/j.amc.2017.05.020

[24] K. Uchizawa, T. Aoki, T. Ito, A. Suzuki and X. Zhou, *On the rainbow connectivity of graphs: complexity and FPT algorithms*, Algorithmica **67** (2013) 161–179.
doi:10.1007/s00453-012-9689-4

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