

RAINBOW TOTAL-COLORING OF COMPLEMENTARY GRAPHS AND ERDŐS-GALLAI TYPE PROBLEM FOR THE RAINBOW TOTAL-CONNECTION NUMBER

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Abstract

A total-colored graph G is rainbow total-connected if any two vertices of G are connected by a path whose edges and internal vertices have distinct colors. The rainbow total-connection number, denoted by $rtc(G)$, of a graph G is the minimum number of colors needed to make G rainbow total-connected. In this paper, we prove that $rtc(G)$ can be bounded by a constant 7 if the following three cases are excluded: $diam(\overline{G}) = 2$, $diam(\overline{G}) = 3$, $\overline{G}$ contains exactly two connected components and one of them is a trivial graph. An example is given to show that this bound is best possible. We also study Erdős-Gallai type problem for the rainbow total-connection number, and compute the lower bounds and precise values for the function $f(n, k)$,

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where $f(n, k)$ is the minimum value satisfying the following property: if $|E(G)| \geq f(n, k)$, then $rtc(G) \leq k$.

Keywords: Rainbow total-coloring, rainbow total-connection number, complementary graph, Erdős-Gallai type problem.

2010 Mathematics Subject Classification: 05C15, 05C35, 05C75.

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Received 15 December 2016

Revised 18 April 2017

Accepted 18 April 2017