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# ANTIPODAL EDGE-COLORINGS OF HYPERCUBES

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**Abstract**

Two vertices of the $k$-dimensional hypercube $Q_k$ are *antipodal* if they differ in every coordinate. Edges $uv$ and $xy$ are *antipodal* if $u$ is antipodal to $x$ and $v$ is antipodal to $y$. An *antipodal edge-coloring* of $Q_k$ is a 2-edge-coloring such that antipodal edges always have different colors. Norine conjectured that for $k \geq 2$, in every antipodal edge-coloring of $Q_k$ some two antipodal vertices are connected by a monochromatic path. Feder and Subi proved this for $k \leq 5$. We prove it for $k \leq 6$.

**Keywords:** antipodal edge-coloring, hypercube, monochromatic geodesic.

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