

## ETERNAL $m$ -SECURITY BONDAGE NUMBERS IN GRAPHS

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### Abstract

An eternal  $m$ -secure set of a graph  $G = (V, E)$  is a set  $S_0 \subseteq V$  that can defend against any sequence of single-vertex attacks by means of multiple guard shifts along the edges of  $G$ . The eternal  $m$ -security number  $\sigma_m(G)$  is the minimum cardinality of an eternal  $m$ -secure set in  $G$ . The eternal  $m$ -security bondage number  $b_{\sigma_m}(G)$  of a graph  $G$  is the minimum cardinality of a set of edges of  $G$  whose removal from  $G$  increases the eternal  $m$ -security number of  $G$ . In this paper, we study properties of the eternal  $m$ -security bondage number. In particular, we present some upper bounds on the eternal  $m$ -security bondage number in terms of eternal  $m$ -security number and edge connectivity number, and we show that the eternal  $m$ -security bondage number of trees is at most 2 and we classify all trees attaining this bound.

**Keywords:** eternal  $m$ -secure set, eternal  $m$ -security number, eternal  $m$ -security bondage number.

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