

MAKING A DOMINATING SET OF A GRAPH CONNECTED

HENGZHE LI

College of Mathematics and Information Science
Henan Normal University, Xinxiang 453007, P.R. China

e-mail: lhz@htu.cn

BAOYINDURENG WU

College of Mathematics and System Sciences
Xinjiang University, Urumqi 830046, P.R. China

e-mail: baoywu@163.com

AND

WEIHUA YANG

School of Mathematical Science
Taiyuan University of Technology
Shanxi Taiyuan 030024, P.R. China

e-mail: ywh222@163.com

Abstract

Let $G = (V, E)$ be a graph and $S \subseteq V$. We say that S is a dominating set of G , if each vertex in $V \setminus S$ has a neighbor in S . Moreover, we say that S is a connected (respectively, 2-edge connected or 2-connected) dominating set of G if $G[S]$ is connected (respectively, 2-edge connected or 2-connected). The domination (respectively, connected domination, or 2-edge connected domination, or 2-connected domination) number of G is the cardinality of a minimum dominating (respectively, connected dominating, or 2-edge connected dominating, or 2-connected dominating) set of G , and is denoted $\gamma(G)$ (respectively $\gamma_1(G)$, or $\gamma'_2(G)$, or $\gamma_2(G)$). A well-known result of Duchet and Meyniel states that $\gamma_1(G) \leq 3\gamma(G) - 2$ for any connected graph G . We show that if $\gamma(G) \geq 2$, then $\gamma'_2(G) \leq 5\gamma(G) - 4$ when G is a 2-edge connected graph and $\gamma_2(G) \leq 11\gamma(G) - 13$ when G is a 2-connected triangle-free graph.

Keywords: independent set, dominating set, connected dominating set.

2010 Mathematics Subject Classification: 05C69.

REFERENCES

- [1] L. Arseneau, A. Finbow, B. Hartnell, A. Hynick, D. MacLean and L. O’Sullivan, *On minimal connected dominating sets*, J. Combin. Math. Combin. Comput. **24** (1997) 185–191.
- [2] C. Bo and B. Liu, *Some inequalities about the connected domination number*, Discrete Math. **159** (1996) 241–245.
doi:10.1016/0012-365X(95)00088-E
- [3] J.A. Bondy and U.S.R. Murty, Graph Theory (GTM 244, Springer, London, 2008).
- [4] M. Chellali, O. Favaron, A. Hansberg and L. Volkmann, *k-domination and k-independence in graphs: A survey*, Graphs Combin. **28** (2012) 1–55.
doi:10.1007/s00373-011-1040-3
- [5] C.J. Colbourn and L.K. Stewart, *Permutation graphs: Connected domination and Steiner trees*, Discrete Math. **86** (1990) 179–189.
doi:10.1016/0012-365X(90)90359-P
- [6] D.-Z. Du and P.-J. Wan, Connected Dominating Set: Theory and Applications (Springer, New York, 2013).
doi:10.1007/978-1-4614-5242-3
- [7] Y.L. Du and H.W. Du, *A new bound on maximum independent set and minimum connected dominating set in unit disk graphs*, J. Comb. Optim. **30** (2015) 1173–1179.
doi:10.1007/s10878-013-9690-0
- [8] P. Duchet and H. Meyniel, *On Hadwiger’s number and the stability number*, North-Holland Math. Studies **62** (1982) 71–73.
doi:10.1016/S0304-0208(08)73549-7
- [9] S. Guha and S. Khuller, *Approximation algorithms for connected dominating sets*, Algorithmica **20**(1998) 374–387.
doi:10.1007/PL00009201
- [10] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: The Theory (Marcel Dekker, New York, 1997).
- [11] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Selected Topics (Marcel Dekker, New York, 1997).
- [12] M. Li, P.-J. Wan and F. Yao, *Tighter approximation bounds for minimum CDS in unit disk graphs*, Algorithmica **61** (2011) 1000–1021.
doi:10.1007/s00453-011-9512-7
- [13] X. Li and Z. Zhang, *Two algorithms for minimum 2-connected r-hop dominating set*, Inform. Process. Lett. **110** (2010) 986–991.
doi:10.1016/j.ipl.2010.08.008
- [14] M. Moscarini, *Doubly chordal graphs, Steiner trees, and connected domination*, Networks **23** (1993) 59–69.
doi:10.1002/net.3230230108

- [15] E. Sampathkumar and H.B. Walikar, *The connected domination number of a graphs*, J. Math. Phys. Sci. **13** (1979) 607–613.
- [16] E. Vigoda, Lecture Notes on a Parallel Algorithm for Generating a Maximal Independent Set, Georgia Institute of Technology, last updated for 7530 – Randomized Algorithms, Spring 2010.
- [17] P.-J. Wan, L. Wang and F. Yao, *Two-phased approximation algorithms for minimum CDS in wireless ad hoc networks*, in: IEEE ICDCS (2008) 337–344.
doi:10.1109/ICDCS.2008.15
- [18] K. White, M. Farber and W.R. Pulleyblank, *Steiner trees, connected domination and strongly chordal graphs*, Networks **15** (1985) 109–124.
doi:10.1002/net.3230150109
- [19] W. Wu, H. Du, X. Jia, Y. Li and S. Huang, *Minimum connected dominating sets and maximal independent sets in unit disk graphs*, Theoret. Comput. Sci. **352** (2006) 1–7.
doi:10.1016/j.tcs.2005.08.037
- [20] W. Wu, X. Gao, P.M. Pardalos and D.-Z. Du, *Wireless networking, dominating and packing*, Optim. Lett. **4** (2010) 347–358.
doi:10.1007/s11590-009-0151-8

Received 20 February 2016

Revised 3 March 2017

Accepted 8 March 2017