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THE SUPER-CONNECTIVITY OF KNESER GRAPHS

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Abstract

A vertex cut of a connected graph G is a set of vertices whose deletion disconnects G. A connected graph G is super-connected if the deletion of every minimum vertex cut of G isolates a vertex. The super-connectivity is the size of the smallest vertex cut of G such that each resultant component does not have an isolated vertex. The Kneser graph $KG(n, k)$ is the graph whose vertices are the k-subsets of $\{1, 2, \ldots, n\}$ and two vertices are adjacent if the k-subsets are disjoint. We use Baranyai's Theorem on the decompositions of complete hypergraphs to show that the Kneser graph $KG(n, 2)$ are super-connected when $n \geq 5$ and that their super-connectivity is $\binom{n}{2} - 6$ when $n \geq 6$.

Keywords: connectivity, super-connectivity, Kneser graphs.

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