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# FACIAL INCIDENCE COLORINGS OF EMBEDDED MULTIGRAPHS [1]

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**Abstract**

Let $G$ be a cellular embedding of a multigraph in a 2-manifold. Two distinct edges $e_1, e_2 \in E(G)$ are facially adjacent if they are consecutive on a facial walk of a face $f \in F(G)$. An incidence of the multigraph $G$ is a pair $(v, e)$, where $v \in V(G)$, $e \in E(G)$ and $v$ is incident with $e$ in $G$. Two distinct incidences $(v_1, e_1)$ and $(v_2, e_2)$ of $G$ are facially adjacent if either $e_1 = e_2$ or $e_1, e_2$ are facially adjacent and either $v_1 = v_2$ or $v_1 \neq v_2$ and there is $i \in \{1, 2\}$ such that $e_i$ is incident with both $v_1, v_2$. A facial incidence coloring of $G$ assigns a color to each incidence of $G$ in such a way that facially adjacent incidences get distinct colors. In this note we show that any embedded multigraph has a facial incidence coloring with seven colors. This bound is improved to six for several wide families of plane graphs and to four for plane triangulations.

**Keywords:** embedded multigraph, incidence, facial incidence coloring.

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