

EQUITABLE COLORING AND EQUITABLE
CHOOSABILITY OF GRAPHS WITH SMALL
MAXIMUM AVERAGE DEGREE¹

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Abstract

A graph is said to be equitably k -colorable if the vertex set $V(G)$ can be partitioned into k independent subsets V_1, V_2, \dots, V_k such that $||V_i| - |V_j|| \leq 1$ ($1 \leq i, j \leq k$). A graph G is equitably k -choosable if, for any given k -uniform list assignment L , G is L -colorable and each color appears on at most $\lceil \frac{|V(G)|}{k} \rceil$ vertices. In this paper, we prove that if G is a graph such that $mad(G) < 3$, then G is equitably k -colorable and equitably k -choosable where $k \geq \max\{\Delta(G), 4\}$. Moreover, if G is a graph such that $mad(G) < \frac{12}{5}$, then G is equitably k -colorable and equitably k -choosable where $k \geq \max\{\Delta(G), 3\}$.

Keywords: graph coloring, equitable choosability, maximum average degree.

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