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FACIAL RAINBOW COLORING OF PLANE GRAPHS

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Abstract

A vertex coloring of a plane graph G is a *facial rainbow coloring* if any two vertices of G connected by a facial path have distinct colors. The *facial rainbow number* of a plane graph G, denoted by $rb(G)$, is the minimum number of colors that are necessary in any facial rainbow coloring of G. Let $L(G)$ denote the order of a longest facial path in G. In the present note we prove that $rb(T) \leq \left\lfloor \frac{3}{2} L(T) \right\rfloor$ for any tree T and $rb(G) \leq \left\lceil \frac{5}{3} L(G) \right\rceil$ for arbitrary simple graph G. The upper bound for trees is tight. For any simple 3-connected plane graph G we have $rb(G) \leq L(G) + 5$.

Keywords: cyclic coloring, rainbow coloring, plane graphs.

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