

TURÁN FUNCTION AND H -DECOMPOSITION PROBLEM FOR GEM GRAPHS

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Abstract

Given a graph H , the *Turán function* $\text{ex}(n, H)$ is the maximum number of edges in a graph on n vertices not containing H as a subgraph. For two graphs G and H , an H -*decomposition* of G is a partition of the edge set of G such that each part is either a single edge or forms a graph isomorphic to H . Let $\phi(n, H)$ be the smallest number ϕ such that any graph G of order n admits an H -decomposition with at most ϕ parts. Pikhurko and Sousa conjectured that $\phi(n, H) = \text{ex}(n, H)$ for $\chi(H) \geq 3$ and all sufficiently large n . Their conjecture has been verified by Özkahya and Person for all edge-critical graphs H . In this article, we consider the *gem graphs* gem_4 and gem_5 . The graph gem_4 consists of the path P_4 with four vertices a, b, c, d and edges ab, bc, cd plus a universal vertex u adjacent to a, b, c, d , and the graph gem_5 is similarly defined with the path P_5 on five vertices. We determine

the Turán functions $\text{ex}(n, \text{gem}_4)$ and $\text{ex}(n, \text{gem}_5)$, and verify the conjecture of Pikhurko and Sousa when H is the graph gem_4 and gem_5 .

Keywords: gem graph, Turán function, extremal graph, graph decomposition.

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