

TREES WITH UNIQUE LEAST CENTRAL SUBTREES

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Abstract

A subtree S of a tree T is a central subtree of T if S has the minimum eccentricity in the join-semilattice of all subtrees of T . Among all subtrees lying in the join-semilattice center, the subtree with minimal size is called the least central subtree. Hamina and Peltola asked what is the characterization of trees with unique least central subtree? In general, it is difficult to characterize completely the trees with unique least central subtree. Nieminen and Peltola [*The subtree center of a tree*, Networks 34 (1999) 272–278] characterized the trees with the least central subtree consisting just of a single vertex. This paper characterizes the trees having two adjacent vertices as a unique least central subtree.

Keywords: tree, central subtree, least central subtree.

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