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$\mathcal{P}$-APEX GRAPHS

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Dedicated to the memory
of Professor Horst Sachs (1927 – 2017)

Abstract

Let $\mathcal{P}$ be an arbitrary class of graphs that is closed under taking induced subgraphs and let $\mathcal{C}(\mathcal{P})$ be the family of forbidden subgraphs for $\mathcal{P}$. We investigate the class $\mathcal{P}(k)$ consisting of all the graphs G for which the removal of no more than k vertices results in graphs that belong to $\mathcal{P}$. This approach provides an analogy to apex graphs and apex-outerplanar graphs studied previously. We give a sharp upper bound on the number of vertices of graphs in $\mathcal{C}(\mathcal{P}(1))$ and we give a construction of graphs in $\mathcal{C}(\mathcal{P}(k))$ of relatively large order for $k \geq 2$. This construction implies a lower bound on the maximum order of graphs in $\mathcal{C}(\mathcal{P}(k))$. Especially, we investigate $\mathcal{C}(\mathcal{W}_r(1))$, where $\mathcal{W}_r$ denotes the class of P_r-free graphs. We determine some forbidden subgraphs for the class $\mathcal{W}_r(1)$ with the minimum and maximum number of vertices. Moreover, we give sufficient conditions for graphs belonging to $\mathcal{C}(\mathcal{P}(k))$, where $\mathcal{P}$ is an additive class, and a characterisation of all forests in $\mathcal{C}(\mathcal{P}(k))$. Particularly we deal with $\mathcal{C}(\mathcal{P}(1))$, where $\mathcal{P}$ is a class closed under substitution and obtain a characterisation of all graphs in the corresponding $\mathcal{C}(\mathcal{P}(1))$. In order to obtain desired results we exploit some hypergraph tools and this technique gives a new result in the hypergraph theory.

Keywords: induced hereditary classes of graphs, forbidden subgraphs, hypergraphs, transversal number.

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References

[1] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer, 2008).

[2] M. Borowiecki and P. Mihók, *Hereditary properties of graphs*, in: V.R. Kulli, Ed., Advances in Graph Theory (Vishawa International Publication Gulbarga, 1991).

[3] S. Cichacz, A. Görlich, M. Zwonek and A. Żak, *On* $(C_n; k)$ *stable graphs*, Electron J. Combin. **18** (2011) #P205.

[4] E. Drgas-Burchardt, *Forbidden graphs for classes of split-like graphs*, European J. Combin. **39** (2014) 68–79.
doi:10.1016/j.ejc.2013.12.004

[5] E. Drgas-Burchardt, *On prime inductive classes of graphs*, European J. Combin. **32** (2011) 1317–1328.
doi:10.1016/j.ejc.2011.05.001

[6] A. Dudek, A. Szymański and M. Zwonek, (H, k) *stable graphs with minimum size*, Discuss. Math. Graph Theory **28** (2008) 137–149.
doi:10.7151/dmgt.1397

[7] A. Dudek and M. Zwonek, (H, k) *stable bipartite graphs with minimum size*, Discuss. Math. Graph Theory **29** (2009) 573–581.
doi:10.7151/dmgt.1465

[8] A. Dudek and A. Żak, *On vertex stability with regard to complete bipartite subgraphs*, Discuss. Math. Graph Theory **30** (2010) 663–669.
doi:10.7151/dmgt.1521

[9] S. Dziobak, Excluded-minor characterization of apex-outerplanar graphs, PhD Thesis (Louisiana State Univ., 2011).

[10] J.-L. Fouquet, H. Thuillier, J.-M. Vanherpe and A.P. Wojda, *On* $(K_q; k)$ *vertex stable graphs with minimum size*, Discrete Math. **312** (2012) 2109–2118.
doi:10.1016/j.disc.2011.04.017

[11] J.-L. Fouquet, H. Thuillier, J.-M. Vanherpe and A.P. Wojda, *On* $(K_q; k)$ *stable graphs with small* k, Electron. J. Combin. **19** (2012) #P50.

[12] P. Frankl and G.Y. Katona, *Extremal* k*-edge-Hamiltonian hypergraphs*, Discrete Math. **308** (2008) 1415–1424.
doi:10.1016/j.disc.2007.07.074

[13] T. Gallai, *Transitiv orientierbare Graphen*, Acta Math. Academiae Scientiarum Hungaricae **18** (1967) 25–66.
doi:10.1007/BF02020961

[14] V. Giakoumakis, *On the closure of graphs under substitution*, Discrete Math. **177** (1997) 83–97.
doi:10.1016/S0012-365X(96)00358-5

[15] A. Gyárfás, J. Lehel and Zs. Tuza, *Upper bound on the order of* τ*-critical hypergraphs*, J. Combin. Theory Ser. B **33** (1982) 161–165.
doi:10.1016/0095-8956(82)90065-X

[16] I. Horváth and G.Y. Katona, *Extremal P_4-stable graphs*, Discrete Appl. Math. **159** (2011) 1786–1792.
doi:10.1016/j.dam.2010.11.016

[17] J.M. Lewis and M. Yannakakis, *The node-deletion problem for hereditary properties is NP-complete*, J. Comput. System Sci. **20** (1980) 219–230.
doi:10.1016/0022-0000(80)90060-4

[18] B. Mohar, *Face covers and the genus problem for apex graphs*, J. Combin. Theory Ser. B **82** (2001) 102–117.
doi:10.1006/jctb.2000.2026

[19] N. Robertson and P. Seymour, *Graph minors XX. Wagner's conjecture*, J. Combin. Theory Ser. B **92** (2004) 325–357.
doi:10.1016/j.jctb.2004.08.001

[20] D.M. Thilikos and H.L. Bodlaender, *Fast partitioning l-apex graphs with applications to approximating maximum induced-subgraph problems*, Inform. Process. Lett. **61** (1997) 227–232.
doi:10.1016/S0020-0190(97)00024-0

[21] Zs. Tuza, *Critical hypergraphs and intersecting set-pair systems*, J. Combin. Theory, Ser. B **39** (1985) 134–145.
doi:10.1016/0095-8956(85)90043-7

[22] H. Whitney, *Congruent graphs and the connectivity of graphs*, Amer. J. Math. **54** (1932) 150–168.
doi:10.2307/2371086

[23] A. Żak, *On $(Kq;k)$-stable graphs*, J. Graph Theory **74** (2013) 216–221.
doi:10.1002/jgt.21705

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