

CONFLICT-FREE CONNECTIONS OF GRAPHS

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Abstract

An edge-colored graph G is conflict-free connected if any two of its vertices are connected by a path, which contains a color used on exactly one of its edges. In this paper the question for the smallest number of colors needed for a coloring of edges of G in order to make it conflict-free connected is investigated. We show that the answer is easy for 2-edge-connected graphs and very difficult for other connected graphs, including trees.

Keywords: edge-coloring, conflict-free connection, 2-edge-connected graph, tree.

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