

## ON THE BETA-NUMBER OF FORESTS WITH ISOMORPHIC COMPONENTS

RIKIO ICHISHIMA

*College of Humanities and Sciences, Nihon University, 3-25-40  
Sakurajousui Setagaya-ku, Tokyo 156-8550, Japan*

**e-mail:** ichishima@msh.biglobe.ne.jp

SUSANA-CLARA LÓPEZ

*Departament de Matemàtiques  
Universitat Politècnica de Catalunya, BarcelonaTech  
C/Esteve Terrades 5, 08860 Castelldefels, Spain*

**e-mail:** susana.clara.lopez@upc.edu

FRANCESC ANTONI MUNTANER-BATLE

AND

AKITO OSHIMA

*Graph Theory and Applications Research Group  
School of Electrical Engineering and Computer Science  
Faculty of Engineering and Built Environment  
The University of Newcastle, NSW 2308 Australia*

**e-mail:** famb1es@yahoo.es  
akitoism@yahoo.co.jp

### Abstract

The beta-number,  $\beta(G)$ , of a graph  $G$  is defined to be either the smallest positive integer  $n$  for which there exists an injective function  $f : V(G) \rightarrow \{0, 1, \dots, n\}$  such that each  $uv \in E(G)$  is labeled  $|f(u) - f(v)|$  and the resulting set of edge labels is  $\{c, c+1, \dots, c + |E(G)|-1\}$  for some positive integer  $c$  or  $+\infty$  if there exists no such integer  $n$ . If  $c = 1$ , then the resulting beta-number is called the strong beta-number of  $G$  and is denoted by  $\beta_s(G)$ . In this paper, we show that if  $G$  is a bipartite graph and  $m$  is odd, then  $\beta(mG) \leq m\beta(G) + m - 1$ . This leads us to conclude that  $\beta(mG) = m|V(G)| - 1$  if  $G$  has the additional property that  $G$  is a graceful nontrivial

tree. In addition to these, we examine the (strong) beta-number of forests whose components are isomorphic to either paths or stars.

**Keywords:** beta-number, strong beta-number, graceful labeling, Skolem sequence, hooked Skolem sequence.

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