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A NOTE ON THE THUE CHROMATIC NUMBER OF LEXICOGRAPHIC PRODUCTS OF GRAPHS

Iztok Peterin

Institute of Mathematics and Physics
Faculty of Electrical Engineering and Computer Science
University of Maribor, Maribor, Slovenia

e-mail: iztok.peterin@um.si

Jens Schreyer

Institute of Mathematics
Faculty of Mathematics and Natural Sciences
Technical University Ilmenau, Ilmenau, Germany

e-mail: jens.schreyer@tu-ilmenau.de

Erika Fecková Škrabuľáková

Institute of Control and Informatization of Production Processes
Faculty of Mining, Ecology, Process Control and Geotechnology
Technical University of Košice, Košice, Slovakia

e-mail: erika.skrabulakova@tuke.sk

and

Andrej Taranenko

Department of Mathematics and Computer Science
Faculty of Natural Sciences and Mathematics
University of Maribor, Maribor, Slovenia

e-mail: andrej.taranenko@um.si

Abstract

A sequence is called *non-repetitive* if none of its subsequences forms a repetition (a sequence $r_1r_2\cdots r_{2n}$ such that $r_i = r_{n+i}$ for all $1 \leq i \leq n$). Let G be a graph whose vertices are coloured. A colouring φ of the graph G is *non-repetitive* if the sequence of colours on every path in G is non-repetitive. The *Thue chromatic number*, denoted by $\pi(G)$, is the minimum number of colours of a non-repetitive colouring of G.

In this short note we present two general upper bounds for the Thue chromatic number for the lexicographic product $G \circ H$ of graphs G and H with respect to some properties of the factors. One upper bound is then used to derive the exact values for $\pi(G \circ H)$ when G is a complete multipartite graph and H an arbitrary graph.

Keywords: non-repetitive colouring, Thue chromatic number, lexicographic product of graphs.

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