EXTREMAL IRREGULAR DIGRAPHS

Joanna Górska, Zdzisław Skupień

AGH Kraków
al. Mickiewicza 30, 30–059 Kraków, Poland

e-mail: gorska@agh.edu.pl
skupien@agh.edu.pl

Zyta Dziechcińska-Halamoda, Zofia Majcher

and

Jerzy Michael

Institute of Mathematics and Informatics
Opole University
ul. Oleska 48, 45–052 Opole, Poland

e-mail: zdziech@uni.opole.pl
majcher@math.uni.opole.pl
michael@uni.opole.pl

Abstract

A digraph is called irregular if its distinct vertices have distinct degree pairs. An irregular digraph is called minimal (maximal) if the removal of any arc (addition of any new arc) results in a non-irregular digraph. It is easily seen that the minimum sizes among irregular n-vertex whether digraphs or oriented graphs are the same and are asymptotic to $(\sqrt{2}/3)\, n^{3/2}$; maximum sizes, however, are asymptotic to n^2 and $n^2/2$, respectively. Let s stand for the sum of initial positive integers, $s = 1, 3, 6, \ldots$. An oriented graph H_s and a digraph F_s, both large (in terms of the size), minimal irregular, and on any such s vertices, $s \geq 21$, are constructed in [*Large minimal irregular digraphs*, Opuscula Math. 23 (2003) 21–24], co-authored by Z. D-H. and three more of the present co-authors (Z.M., J.M., Z.S.). In the present paper we nearly complete these constructions. Namely, a large minimal irregular digraph F_n, respectively oriented graph H_n, are constructed for any of remaining orders n, $n > 21$, and of size asymptotic to n^2, respectively to $n^2/2$. Also a digraph Φ_n and an oriented graph G_n, both small maximal irregular of any order $n \geq 6$, are constructed. The asymptotic value of the size of G_n is at least

$(\sqrt{2}/3)\,n^{3/2}$ and is just the least if $n = s \to \infty$, but otherwise the value is at most four times larger and is just the largest if $n = s - 1 \to \infty$. On the other hand, the size of Φ_n is of the asymptotic order $\Theta(n^{3/2})$.

REFERENCES

[1] G. Chartrand and L. Lesniak, Graphs & Digraphs, 3rd Edition (Chapman & Hall, 1996).

[2] Z. Dziechcińska-Halamoda, Z. Majcher, J. Michael and Z. Skupień, *Large minimal irregular digraphs*, Opuscula Math. **23** (2003) 21–24.

[3] Z. Dziechcińska-Halamoda, Z. Majcher, J. Michael and Z. Skupień, *Extremum degree sets of irregular oriented graphs and pseudodigraphs*, Discuss. Math. Graph Theory **26** (2006) 317–333.
doi:10.7151/dmgt.1323

[4] M. Gargano, J.W. Kennedy and L.V. Quintas, *Irregular digraphs*, Congr. Numer. **72** (1990) 223–231.

[5] J. Górska, Z. Skupień, Z. Majcher and J. Michael, *A smallest irregular oriented graph containing a given diregular one*, Discrete Math. **286** (2004) 79–88.
doi:10.1016/j.disc.2003.11.049

[6] Z. Majcher, J. Michael, J. Górska and Z. Skupień, *The minimum size of fully irregular oriented graphs*, Discrete Math. **236** (2001) 263–272.
doi:10.1016/S0012-365X(00)00446-5

[7] Z. Skupień, *Problems on fully irregular digraphs*, in: Z. Skupień and R. Kalinowski, guest eds., Discuss. Math. Graph Theory **19** (1999) 253–255.
doi:10.7151/dmgt.1102