# IRREDUCIBLE NO-HOLE $L(2,1)$-COLORING OF EDGE-MULTIPLICITY-PATHS-REPLACEMENT GRAPH 

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#### Abstract

An $L(2,1)$-coloring (or labeling) of a simple connected graph $G$ is a mapping $f: V(G) \rightarrow Z^{+} \cup\{0\}$ such that $|f(u)-f(v)| \geq 2$ for all edges $u v$ of $G$, and $|f(u)-f(v)| \geq 1$ if $u$ and $v$ are at distance two in $G$. The span of an $L(2,1)$-coloring $f$, denoted by $\operatorname{span}(f)$, of $G$ is $\max \{f(v): v \in V(G)\}$. The span of $G$, denoted by $\lambda(G)$, is the minimum span of all possible $L(2,1)$ colorings of $G$. For an $L(2,1)$-coloring $f$ of a graph $G$ with span $k$, an integer $l$ is a hole in $f$ if $l \in(0, k)$ and there is no vertex $v$ in $G$ such that $f(v)=l$. An $L(2,1)$-coloring is a no-hole coloring if there is no hole in it, and is an irreducible coloring if color of none of the vertices in the graph can be decreased and yield another $L(2,1)$-coloring of the same graph. An irreducible no-hole coloring, in short inh-coloring, of $G$ is an $L(2,1)$-coloring of $G$ which is both irreducible and no-hole. For an inh-colorable graph $G$, the inh-span of $G$, denoted by $\lambda_{i n h}(G)$, is defined as $\lambda_{i n h}(G)=\min \{\operatorname{span}(f): f$ is an inh-coloring of $G\}$. Given a function $h: E(G) \rightarrow \mathbb{N}-\{1\}$, and a positive integer $r \geq 2$, the edge-multiplicity-paths-replacement graph $G\left(r P_{h}\right)$ of $G$ is the graph obtained by replacing every edge $u v$ of $G$ with $r$ paths of length $h(u v)$ each. In this paper we show that $G\left(r P_{h}\right)$ is inh-colorable except possibly the cases $h(e) \geq 2$ with equality for at least one but not for all edges $e$ and (i) $\Delta(G)=2, r=2$ or (ii) $\Delta(G) \geq 3,2 \leq r \leq 4$. We find the exact value of $\lambda_{i n h}\left(G\left(r P_{h}\right)\right)$ in several cases and give upper bounds of the same in the remaining. Moreover, we find the value of $\lambda\left(G\left(r P_{h}\right)\right)$ in most of the cases which were left by Lü and Sun in $[L(2,1)$-labelings of the edge-multiplicity-paths-replacement of a graph, J. Comb. Optim. 31 (2016) 396-404].


Keywords: $L(2,1)$-coloring, no-hole coloring, irreducible coloring, subdivision graph, edge-multiplicity-paths-replacement graph.
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