

IRREDUCIBLE NO-HOLE $L(2, 1)$ -COLORING OF EDGE-MULTIPLICITY-PATHS-REPLACEMENT GRAPH

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Abstract

An $L(2, 1)$ -coloring (or *labeling*) of a simple connected graph G is a mapping $f : V(G) \rightarrow \mathbb{Z}^+ \cup \{0\}$ such that $|f(u) - f(v)| \geq 2$ for all edges uv of G , and $|f(u) - f(v)| \geq 1$ if u and v are at distance two in G . The *span of an $L(2, 1)$ -coloring f* , denoted by $\text{span}(f)$, of G is $\max\{f(v) : v \in V(G)\}$. The *span of G* , denoted by $\lambda(G)$, is the minimum span of all possible $L(2, 1)$ -colorings of G . For an $L(2, 1)$ -coloring f of a graph G with span k , an integer l is a *hole* in f if $l \in (0, k)$ and there is no vertex v in G such that $f(v) = l$. An $L(2, 1)$ -coloring is a *no-hole coloring* if there is no hole in it, and is an *irreducible coloring* if color of none of the vertices in the graph can be decreased and yield another $L(2, 1)$ -coloring of the same graph. An *irreducible no-hole coloring*, in short *inh-coloring*, of G is an $L(2, 1)$ -coloring of G which is both irreducible and no-hole. For an inh-colorable graph G , the *inh-span* of G , denoted by $\lambda_{inh}(G)$, is defined as $\lambda_{inh}(G) = \min\{\text{span}(f) : f \text{ is an inh-coloring of } G\}$. Given a function $h : E(G) \rightarrow \mathbb{N} - \{1\}$, and a positive integer $r \geq 2$, the *edge-multiplicity-paths-replacement graph $G(rP_h)$* of G is the graph obtained by replacing every edge uv of G with r paths of length $h(uv)$ each. In this paper we show that $G(rP_h)$ is inh-colorable except possibly the cases $h(e) \geq 2$ with equality for at least one but not for all edges e and (i) $\Delta(G) = 2$, $r = 2$ or (ii) $\Delta(G) \geq 3$, $2 \leq r \leq 4$. We find the exact value of $\lambda_{inh}(G(rP_h))$ in several cases and give upper bounds of the same in the remaining. Moreover, we find the value of $\lambda(G(rP_h))$ in most of the cases which were left by Lü and Sun in [*$L(2, 1)$ -labelings of the edge-multiplicity-paths-replacement of a graph*, J. Comb. Optim. 31 (2016) 396–404].

Keywords: $L(2, 1)$ -coloring, no-hole coloring, irreducible coloring, subdivision graph, edge-multiplicity-paths-replacement graph.

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