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TOTAL DOMINATION VERSUS PAIRED-DOMINATION IN REGULAR GRAPHS

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Abstract

A subset S of vertices of a graph G is a dominating set of G if every vertex not in S has a neighbor in S, while S is a total dominating set of G if every vertex has a neighbor in S. If S is a dominating set with the additional property that the subgraph induced by S contains a perfect matching, then S is a paired-dominating set. The domination number, denoted $\gamma(G)$, is the minimum cardinality of a dominating set of G, while the minimum cardinalities of a total dominating set and paired-dominating set are the total domination number, $\gamma_t(G)$, and the paired-domination number, $\gamma_{\rm pr}(G)$, respectively. For $k \geq 2$, let G be a connected k-regular graph. It is known [Schaudt, *Total domination versus paired domination*, Discuss. Math. Graph Theory **32** (2012) 435–447] that $\gamma_{\rm pr}(G)/\gamma_t(G) \leq (2k)/(k+1)$. In the special case when $k = 2$, we observe that $\gamma_{\rm pr}(G)/\gamma_t(G) \leq 4/3$, with equality if and only if $G \cong C_5$. When $k = 3$, we show that $\gamma_{\rm pr}(G)/\gamma_t(G) \leq 3/2$, with equality if and only if G is the Petersen graph. More generally for $k \geq 2$, if G has girth at least 5 and satisfies $\gamma_{\rm pr}(G)/\gamma_t(G) = (2k)/(k+1)$, then we show that G is a diameter-2 Moore graph. As a consequence of this result, we prove that for $k \geq 2$ and $k \neq 57$, if G has girth at least 5, then

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$\gamma_{\rm pr}(G)/\gamma_t(G) \le (2k)/(k+1)$, with equality if and only if $k = 2$ and $G \cong C_5$ or $k = 3$ and G is the Petersen graph.

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