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HOMOMORPHIC PREIMAGES OF GEOMETRIC PATHS

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Abstract

A graph G is a homomorphic preimage of another graph H, or equivalently G is H-colorable, if there exists a graph homomorphism $f : G \to H$. A geometric graph $\overline{G}$ is a simple graph G together with a straight line drawing of G in the plane with the vertices in general position. A geometric homomorphism (respectively, isomorphism) $\overline{G} \to \overline{H}$ is a graph homomorphism (respectively, isomorphism) that preserves edge crossings (respectively, and non-crossings). The homomorphism poset $\mathcal{G}$ of a graph G is the set of isomorphism classes of geometric realizations of G partially ordered by the existence of injective geometric homomorphisms. A geometric graph $\overline{G}$ is $\mathcal{H}$-colorable if $\overline{G} \to \overline{H}$ for some $\overline{H} \in \mathcal{H}$. In this paper, we provide necessary and sufficient conditions for $\overline{G}$ to be $\mathcal{P}_n$-colorable for $n \geq 2$. Along the way, we also provide necessary and sufficient conditions for $\overline{G}$ to be $\mathcal{K}_{2,3}$-colorable.

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