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ARC-DISJOINT HAMILTONIAN CYCLES IN ROUND DECOMPOSABLE LOCALLY SEMICOMPLETE DIGRAPHS

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Abstract

Let $D = (V, A)$ be a digraph; if there is at least one arc between every pair of distinct vertices of D, then D is a semicomplete digraph. A digraph D is locally semicomplete if for every vertex x, the out-neighbours of x induce a semicomplete digraph and the in-neighbours of x induce a semicomplete digraph. A locally semicomplete digraph without 2-cycle is a local tournament. In 2012, Bang-Jensen and Huang [J. Combin Theory Ser. B 102 (2012) 701–714] concluded that every 2-arc-strong locally semicomplete digraph which is not the second power of an even cycle has two arc-disjoint strong spanning subdigraphs, and proposed the conjecture that every 3-strong local tournament has two arc-disjoint Hamiltonian cycles. According to Bang-Jensen, Guo, Gutin and Volkmann, locally semicomplete digraphs have three subclasses: the round decomposable; the non-round decomposable which are not semicomplete; the non-round decomposable which are semicomplete. In this paper, we prove that every 3-strong round decomposable locally semicomplete digraph has two arc-disjoint Hamiltonian cycles, which implies that the conjecture holds for the round decomposable local tournaments. Also, we characterize the 2-strong round decomposable local tournaments each of which contains a Hamiltonian path P and a Hamiltonian cycle arc-disjoint from P.

Keywords: locally semicomplete digraph, local tournament, round decomposable, arc-disjoint, Hamiltonian cycle, Hamiltonian path.

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