

SOME RESULTS ON THE INDEPENDENCE POLYNOMIAL OF UNICYCLIC GRAPHS

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Abstract

Let G be a simple graph on n vertices. An independent set in a graph is a set of pairwise non-adjacent vertices. The independence polynomial of G is the polynomial $I(G, x) = \sum_{k=0}^n s(G, k)x^k$, where $s(G, k)$ is the number of independent sets of G with size k and $s(G, 0) = 1$. A unicyclic graph is a graph containing exactly one cycle. Let C_n be the cycle on n vertices. In this paper we study the independence polynomial of unicyclic graphs. We show that among all connected unicyclic graphs G on n vertices (except two of them), $I(G, t) > I(C_n, t)$ for sufficiently large t . Finally for every $n \geq 3$ we find all connected graphs H such that $I(H, x) = I(C_n, x)$.

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