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A LIMIT CONJECTURE ON THE NUMBER OF HAMILTONIAN CYCLES ON THIN TRIANGULAR GRID CYLINDER GRAPHS [1]

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Abstract

We continue our research in the enumeration of Hamiltonian cycles (HCs) on thin cylinder grid graphs $C_m \times P_{n+1}$ by studying a triangular variant of the problem. There are two types of HCs, distinguished by whether they wrap around the cylinder. Using two characterizations of these HCs, we prove that, for fixed m, the number of HCs of both types satisfy some linear recurrence relations. For small m, computational results reveal that the two

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numbers are asymptotically the same. We conjecture that this is true for all $m \geq 2$.

Keywords: contractible Hamiltonian cycles, generating functions, thin triangular grid cylinder graph.

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