

THE SMALLEST HARMONIC INDEX OF TREES WITH GIVEN MAXIMUM DEGREE

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Abstract

The harmonic index of a graph G , denoted by $H(G)$, is defined as the sum of weights $2/[d(u) + d(v)]$ over all edges uv of G , where $d(u)$ denotes the degree of a vertex u . In this paper we establish a lower bound on the harmonic index of a tree T .

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