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THE SMALLEST HARMONIC INDEX OF TREES WITH GIVEN MAXIMUM DEGREE

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Abstract

The harmonic index of a graph G, denoted by $H(G)$, is defined as the sum of weights $2/[d(u) + d(v)]$ over all edges uv of G, where $d(u)$ denotes the degree of a vertex u. In this paper we establish a lower bound on the harmonic index of a tree T.

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