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THE SMALLEST HARMONIC INDEX OF TREES WITH GIVEN MAXIMUM DEGREE

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Abstract

The harmonic index of a graph G, denoted by $H(G)$, is defined as the sum of weights $2/[d(u) + d(v)]$ over all edges uv of G, where $d(u)$ denotes the degree of a vertex u. In this paper we establish a lower bound on the harmonic index of a tree T.

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References

[1] H. Deng, S. Balachandran, S.K. Ayyaswamy and Y.B. Venkatakrishnan, *On the harmonic index and the chromatic number of a graph*, Discrete Appl. Math. **161** (2013) 2740–2744.
doi:10.1016/j.dam.2013.04.003

[2] H. Deng, S. Balachandran, S.K. Ayyaswamy and V.B. Venkatakrishnan, *On harmonic indices of trees, unicyclic graphs and bicyclic graphs*, Ars Combin. **130** (2017) 239–248.

[3] S. Fajtlowicz, *On conjectures of Graffiti*-II, Congr. Numer. **60** (1987) 187–197.

[4] B. Furtula, I. Gutman and M. Dehmer, *On structure-sensitivity of degree-based topological indices*, Appl. Math. Comput. **219** (2013) 8973–8978.
doi:10.1016/j.amc.2013.03.072

[5] O. Favaron, M. Mahio and J.F. Sacle, *Some eigenvalue properties in graphs (Conjectures of Graffiti*-II), Discrete Math. **111** (1993) 197–220. doi:10.1016/0012-365X(93)90156-N

[6] I. Gutman, *Degree-based topological indices*, Croat. Chem. Acta **86** (2013) 351–361. doi:10.5562/cca2294

[7] I. Gutman and N. Trinajstić, *Graph theory and molecular orbitals,Total-electron energy of alternant hydrocarbons*, Chem. Phys. Lett. **17** (1972) 535–538. doi:10.1016/0009-2614(72)85099-1

[8] I. Gutman, L. Zhong and K. Xu, *Relating ABC and harmonic indices*, J. Serb. Chem. Soc. **79** (2014) 557–563. doi:10.2298/JSC130930001G

[9] Y. Hu and X. Zhou, *On the harmonic index of the unicyclic and bicyclic graphs*, WSEAS Trans. Math. **12** (2013) 716–726.

[10] A. Ilic, *Note on the harmonic index of a graph*, Appl. Math. Lett. **25** (2012) 561–566. doi:10.1016/j.aml.2011.09.059

[11] M.A. Iranmanesh and M. Saheli, *On the harmonic index and harmonic polynomial of caterpillars with diameter four*, Iranian J. Math. Chem. **6** (2015) 41–49.

[12] J. Li and W.C. Shiu, *The harmonic index of a graph*, Rocky Mountain J. Math. **44** (2014) 1607–1620. doi:0.1216/RMJ-2014-44-5-1607

[13] J. Liu, *On harmonic index and diameter of graphs*, J. Appl. Math. Phys. **1** (2013) 5–6. doi:10.4236/jamp.2013.13002

[14] J. Liu, *On the harmonic index of triangle-free graphs*, Appl. Math. **4** (2013) 1204–1206. doi:10.4236/am.2013.48161

[15] J. Liu, *Harmonic index of dense graphs*, Ars Combin. **120** (2015) 293–304.

[16] J. Liu and Q. Zhang, *Remarks on harmonic index of graphs*, Util. Math. **88** (2012) 281–285.

[17] J.B. Lv, J. Li and W.C. Shiu, *The harmonic index of unicyclic graphs with given matching number*, Kragujevac J. Math. **38** (2014) 173–183. doi:doi.org/10.5937/KgJMath1401173J

[18] S. Liu and J. Liu, *Some properties on the harmonic index of molecular trees*, ISRN Appl. Math. (2014) 1–8.

[19] R. Rasi, S.M. Sheikholeslami and I. Gutman, *On harmonic index of trees*, MATCH Commun. Math. Comput. Chem. **78** (2017) 405–416.

[20] R.Wu, Z. Tang and H. Deng, *A lower bound for the harmonic index of a graph with minimum degree at least two*, Filomat **27** (2013) 51–55. doi:10.2298/FIL1301051W

[21] R. Wu, Z. Tang and H. Deng, *On the harmonic index and the girth of a graph*, Util. Math. **91** (2013) 65–69.

[22] X. Xu, *Relationships between harmonic index and other topological indices*, Appl. Math. Sci. **6** (2012) 2013–2018.

[23] L. Yang and H. Hua, *The harmonic index of general graphs, nanocones and triangular benzenoid graphs*, Optoelectron. Adv. Mater. - Rapid Commun. **6** (2012) 660–663.

[24] L. Zhong, *The harmonic index for graphs*, Appl. Math. Lett. **25** (2012) 561–566. doi:10.1016/j.aml.2011.09.059

[25] L. Zhong, *The harmonic index of unicyclic graphs*, Ars Combin. **104** (2012) 261–269.

[26] L. Zhong, *The harmonic index for unicyclic and bicyclic graphs with given matching number*, Miskolc Math. Notes **16** (2015) 587–605.

[27] L. Zhong and Q. Cui, *The harmonic index for unicyclic graphs with given girth*, Filomat **29** (2015) 673–686. doi:10.2298/FIL1504673Z

[28] L. Zhong and K. Xu, *The harmonic index for bicyclic graphs*, Util. Math. **90** (2013) 23–32.

[29] L. Zhong and K. Xu, *Inequalities between vertex-degree-based topological indices*, MATCH Commun. Math. Comput. Chem. **71** (2014) 627–642.

[30] Y. Zhu, R. Chang and X. Wei, *The harmonic index on bicyclic graphs*, Ars Combin. **110** (2013) 97–104.

[31] Y. Zhu and R. Chang, *On the harmonic index of bicyclic conjugated molecular graphs*, Filomat **28** (2014) 421–428. doi:10.2298/FIL1402421Z

[32] Y. Zhu and R. Chang, *Minimum harmonic index of trees and unicyclic graphs with given number of pendant vertices and diameter*, Util. Math. **93** (2014) 345–374.

[33] A. Zolfi, A.R. Ashrafi and S. Moradi, *The top ten values of harmonic index in chemical trees*, Kragujevac J. Sci. **37** (2015) 91–98.

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