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THE SECOND NEIGHBOURHOOD FOR BIPARTITE TOURNAMENTS

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Abstract

Let $T(X \cup Y, A)$ be a bipartite tournament with partite sets X, Y and arc set A. For any vertex $x \in X \cup Y$, the second out-neighbourhood $N^{++}(x)$ of x is the set of all vertices with distance 2 from x. In this paper, we prove that T contains at least two vertices x such that $|N^{++}(x)| \geq |N^{+}(x)|$ unless T is in a special class $\mathcal{B}_1$ of bipartite tournaments; show that T contains at least a vertex x such that $|N^{++}(x)| \geq |N^{-}(x)|$ and characterize the class $\mathcal{B}_2$ of bipartite tournaments in which there exists exactly one vertex x with this property; and prove that if $|X| = |Y|$ or $|X| \geq 4|Y|$, then the bipartite tournament T contains a vertex x such that $|N^{++}(x)|+|N^{+}(x)| \geq 2|N^{-}(x)|$.

Keywords: second out-neighbourhood, out-neighbourhood, in-neighbourhood, bipartite tournament.

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