

BOUNDING THE OPEN k -MONOPOLY NUMBER OF STRONG PRODUCT GRAPHS

DOROTA KUZIAK

Departamento de Estadística e Investigación Operativa

EPS, Universidad de Cádiz

Av. Ramón Puyol s/n, 11202 Algeciras, Spain

e-mail: dorota.kuziak@uca.es

IZTOK PETERIN

University of Maribor, FEECS

Koroška 46, 2000 Maribor, Slovenia

and

IMFM, Jadranska 19, 1000 Ljubljana, Slovenia

e-mail: izard.peterin@um.si

AND

ISMAEL G. YERO

Departamento de Matemáticas, EPS, Universidad de Cádiz

Av. Ramón Puyol s/n, 11202 Algeciras, Spain

e-mail: ismael.gonzalez@uca.es

Abstract

Let $G = (V, E)$ be a simple graph without isolated vertices and minimum degree δ , and let $k \in \{1 - \lceil \delta/2 \rceil, \dots, \lfloor \delta/2 \rfloor\}$ be an integer. Given a set $M \subset V$, a vertex v of G is said to be k -controlled by M if $\delta_M(v) \geq \frac{\delta_G(v)}{2} + k$, where $\delta_M(v)$ represents the number of neighbors of v in M and $\delta_G(v)$ the degree of v in G . A set M is called an open k -monopoly if every vertex v of G is k -controlled by M . The minimum cardinality of any open k -monopoly is the open k -monopoly number of G . In this article we study the open k -monopoly number of strong product graphs. We present general lower and upper bounds for the open k -monopoly number of strong product graphs. Moreover, we study in addition the open 0-monopolies of several specific families of strong product graphs.

Keywords: open monopolies, strong product graphs, alliances, domination.

2010 Mathematics Subject Classification: 05C69, 05C76.

REFERENCES

- [1] D.W. Bange, A.E. Barkauskas and P.J. Slater, *Efficient dominating sets in graphs*, in: Applications of Discrete Math., R.D. Ringeisen and F.S. Roberts (Ed(s)), (SIAM, Philadelphia, 1988) 189–199.
- [2] N. Biggs, *Perfect codes in graphs*, J. Combin. Theory Ser. B **15** (1973) 289–296.
doi:10.1016/0095-8956(73)90042-7
- [3] C. Dwork, D. Peleg, N. Pippenger and E. Upfal, *Fault tolerance in networks of bounded degree*, SIAM J. Comput. **17** (1988) 975–988.
doi:10.1137/0217061
- [4] H. Fernau, J.A. Rodríguez-Velázquez and J.M. Sigarreta, *Global powerful r-alliances and total k-domination in graphs*, Util. Math. **98** (2015) 127–147.
- [5] P. Flocchini, R. Královic, A. Roncato, P. Ružička and N. Santoro, *On time versus size for monotone dynamic monopolies in regular topologies*, J. Discrete Algorithms **1** (2003) 129–150.
doi:10.1016/S1570-8667(03)00022-4
- [6] H. García-Molina and D. Barbara, *How to assign votes in a distributed system*, J. ACM **32** (1985) 841–860.
doi:10.1145/4221.4223
- [7] R. Hammack, W. Imrich and S. Klavžar, Handbook of Product Graphs, Second Edition (CRC Press, Boca Raton, FL, 2011).
- [8] K. Khoshkhah, M. Nemati, H. Soltani and M. Zaker, *A study of monopolies in graphs*, Graphs Combin. **29** (2013) 1417–1427.
doi:10.1007/s00373-012-1214-7
- [9] P. Kristiansen, S.M. Hedetniemi and S.T. Hedetniemi, *Alliances in graphs*, J. Combin. Math. Combin. Comput. **48** (2004) 157–177.
- [10] D. Kuziak, I. Peterin and I.G. Yero, *Computing the (k-)monopoly number of direct product of graphs*, Filomat **29** (2015) 1163–1171.
doi:10.2298/FIL1505163K
- [11] D. Kuziak, I. Peterin and I.G. Yero, *On the monopolies of lexicographic product graphs: bounds and closed formulae*, Bull. Math. Soc. Sci. Math. Roumanie N.S. **59** (2016) 355–366.
- [12] D. Kuziak, I. Peterin and I.G. Yero, *Open k-monopolies in graphs: complexity and related concepts*, Discrete Math. Theor. Comput. Sci. **18** (3) (2016).
- [13] N. Linial, D. Peleg, Yu. Rabinovich and M. Saks, *Sphere packing and local majorities in graphs*, in: Proc. 2nd Israel Symposium on Theory and Computing Systems (Natanya, Israel, 1993) 141–149.
doi:10.1109/ISTCS.1993.253475
- [14] A. Mishra and S.B. Rao, *Minimum monopoly in regular and tree graphs*, Discrete Math. **306** (2006) 1586–1594.
doi:10.1016/j.disc.2005.06.036

- [15] D. Peleg, *Local majorities, coalitions and monopolies in graphs: a review*, Theoret. Comput. Sci. **282** (2002) 231–257.
doi:10.1016/S0304-3975(01)00055-X
- [16] I. Peterin, *The complexity of open k -monopolies in graphs for negative k* , manuscript (2016).
- [17] G.F. Sullivan, Complexity of System-Level Fault Diagnosis and Diagnosability, Ph.D. Thesis (Yale University, New Haven, CT, USA, 1986).
- [18] I.G. Yero and J.A. Rodríguez-Velázquez, *A survey on alliances in graphs: defensive alliances*, Util. Math. (2016), to appear.
- [19] M. Zaker, *On dynamic monopolies of graphs with general thresholds*, Discrete Math. **312** (2012) 1136–1143.
doi:10.1016/j.disc.2011.11.038

Received 26 November 2015

Revised 15 December 2016

Accepted 19 December 2016