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BOUNDING THE OPEN k-MONOPOLY NUMBER OF STRONG PRODUCT GRAPHS

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Abstract

Let $G = (V, E)$ be a simple graph without isolated vertices and minimum degree δ, and let $k \in \{1 - \lceil \delta/2 \rceil, \ldots, \lfloor \delta/2 \rfloor\}$ be an integer. Given a set $M \subset V$, a vertex v of G is said to be k-controlled by M if $\delta_M(v) \geq \frac{\delta_G(v)}{2} + k$, where $\delta_M(v)$ represents the number of neighbors of v in M and $\delta_G(v)$ the degree of v in G. A set M is called an open k-monopoly if every vertex v of G is k-controlled by M. The minimum cardinality of any open k-monopoly is the open k-monopoly number of G. In this article we study the open k-monopoly number of strong product graphs. We present general lower and upper bounds for the open k-monopoly number of strong product graphs. Moreover, we study in addition the open 0-monopolies of several specific families of strong product graphs.

Keywords: open monopolies, strong product graphs, alliances, domination.

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