Discussiones Mathematicae
Graph Theory 38 (2018) 301–317
doi:10.7151/dmgt.2016

ON THE TOTAL k-DOMINATION IN GRAPHS

Sergio Bermudo

Department of Economics
Quantitative Methods and Economic History
Universidad Pablo de Olavide, ES-41013 Seville, Spain
e-mail: sbernav@upo.es

Juan C. Hernández-Gómez

and

José M. Sigarreta

Faculty of Mathematics
Autonomous University of Guerrero
Carlos E. Adame 5, Col. La Garita, Acapulco, Guerrero, Mexico
e-mail: jcarloshg@gmail.com
josemariasigarretaalmira@hotmail.com

Abstract

Let $G = (V, E)$ be a graph; a set $S \subseteq V$ is a total k-dominating set if every vertex $v \in V$ has at least k neighbors in S. The total k-domination number $\gamma_{kt}(G)$ is the minimum cardinality among all total k-dominating sets. In this paper we obtain several tight bounds for the total k-domination number of a graph. In particular, we investigate the relationship between the total k-domination number of a graph and the order, the size, the girth, the minimum and maximum degree, the diameter, and other domination parameters of the graph.

Keywords: k-domination, total k-domination, k-tuple domination, k-tuple total domination.

2010 Mathematics Subject Classification: 05C69.

References

[1] S. Bermudo, J.L. Sánchez and J.M. Sigarreta, *Total k-domination in Cartesian product graphs*, Period. Math. Hungar. (2016), to appear.

[2] S. Bermudo, D.L. Jalemskaya and J.M. Sigarreta, *Total* 2*-domination in grid graphs*, Util. Math. (2017), to appear.

[3] E.J. Cockayne and A.G. Thomason, *An upper bound for the k-tuple domination number*, J. Combin. Math. Combin. Comput. **64** (2008) 251–254.

[4] P. Dorbec, S. Gravier, S. Klavžar and S. Špacapan, *Some results on total domination in direct products of graphs*, Discuss. Math. Graph Theory **26** (2006) 103–112. doi:10.7151/dmgt.1305

[5] O. Favaron, M.A. Henning, J. Puech and D. Rautenbach, *On domination and annihilation in graphs with claw-free blocks*, Discrete Math. **231** (2001) 143–151. doi:10.1016/S0012-365X(00)00313-7

[6] H. Fernau, J.A. Rodríguez-Velázquez and J.M. Sigarreta, *Global powerful* r*-alliances and total* k*-domination in graphs*, Util. Math. **98** (2015) 127–147.

[7] J. Harant and M.A. Henning, *On double domination in graphs*, Discuss. Math. Graph Theory **25** (2005) 29–34. doi:10.7151/dmgt.1256

[8] F. Harary and T.W. Haynes, *Double domination in graphs*, Ars Combin. **55** (2000) 201–213.

[9] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs (Marcel Dekker, Inc., New York, 1998).

[10] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics (Marcel Dekker, Inc., New York, 1998).

[11] M.A. Henning, *A survey of selected recent results on total domination in graphs*, Discrete Math. **309** (2009) 32–63. doi:10.1016/j.disc.2007.12.044

[12] M.A. Henning and A.P. Kazemi, *k-tuple total domination in graphs*, Discrete Appl. Math. **158** (2010) 1006–1011. doi:10.1016/j.dam.2010.01.009

[13] M.A. Henning and A.P. Kazemi, *k-tuple total domination in cross products of graphs*, J. Comb. Optim. **24** (2012) 339–346. doi:10.1007/s10878-011-9389-z

[14] M.A. Henning and A. Yeo, Total Domination in Graphs (Springer Monographs in Mathematics, 2013).

[15] A.P. Kazemi, *On the total k-domination number of graphs*, Discuss. Math. Graph Theory **32** (2012) 419–426. doi:10.7151/dmgt.1616

[16] A. Klobucar, *Total domination numbers of Cartesian products*, Math. Commun. **9** (2004) 35–44.

[17] V.R. Kulli, *On n-total domination number in graphs*, Graph theory, Combinatorics, Algorithms, and Applications (San Francisco, CA, 1989), (SIAM, Philadelphia, PA, 1991) 319–324.

Received 21 April 2016
Revised 21 December 2016
Accepted 21 December 2016