# GENERALIZED RAINBOW CONNECTION OF GRAPHS AND THEIR COMPLEMENTS 

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#### Abstract

Let $G$ be an edge-colored connected graph. A path $P$ in $G$ is called $\ell$-rainbow if each subpath of length at most $\ell+1$ is rainbow. The graph $G$ is called $(k, \ell)$-rainbow connected if there is an edge-coloring such that every pair of distinct vertices of $G$ is connected by $k$ pairwise internally vertex-disjoint $\ell$-rainbow paths in $G$. The minimum number of colors needed to make $G(k, \ell)$-rainbow connected is called the $(k, \ell)$-rainbow connection number of $G$ and denoted by $r c_{k, \ell}(G)$. In this paper, we first focus on the $(1,2)$-rainbow connection number of $G$ depending on some constraints of $\bar{G}$. Then, we characterize the graphs of order $n$ with (1,2)-rainbow connection number $n-1$ or $n-2$. Using this result, we investigate the Nordhaus-Gaddum-Type problem of (1,2)-rainbow connection number and prove that $r c_{1,2}(G)+r c_{1,2}(\bar{G}) \leq n+2$ for connected graphs $G$ and $\bar{G}$. The equality holds if and only if $G$ or $\bar{G}$ is isomorphic to a double star.


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