

## THE COMPLEXITY OF SECURE DOMINATION PROBLEM IN GRAPHS

HAICHAO WANG<sup>1</sup>, YANCAI ZHAO<sup>2</sup>

AND

YUNPING DENG<sup>1</sup>

<sup>1</sup>*Department of Mathematics  
Shanghai University of Electric Power  
Shanghai 200090, China*

<sup>2</sup>*Department of Basic Science  
Wuxi City College of Vocational Technology  
Jiangsu 214153, China*

**e-mail:** whchao2000@163.com  
zhaoyc69@126.com  
dyp612@163.com

### Abstract

A dominating set of a graph  $G$  is a subset  $D \subseteq V(G)$  such that every vertex not in  $D$  is adjacent to at least one vertex in  $D$ . A dominating set  $S$  of  $G$  is called a secure dominating set if each vertex  $u \in V(G) \setminus S$  has one neighbor  $v$  in  $S$  such that  $(S \setminus \{v\}) \cup \{u\}$  is a dominating set of  $G$ . The secure domination problem is to determine a minimum secure dominating set of  $G$ . In this paper, we first show that the decision version of the secure domination problem is NP-complete for star convex bipartite graphs and doubly chordal graphs. We also prove that the secure domination problem cannot be approximated within a factor of  $(1 - \varepsilon) \ln |V|$  for any  $\varepsilon > 0$ , unless  $\text{NP} \subseteq \text{DTIME}(|V|^{O(\log \log |V|)})$ . Finally, we show that the secure domination problem is APX-complete for bounded degree graphs.

**Keywords:** secure domination, star convex bipartite graph, doubly chordal graph, NP-complete, APX-complete.

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