Discussiones Mathematicae
Graph Theory 38 (2018) 385–396
doi:10.7151/dmgt.2008

Full PDF

DMGT Page

THE COMPLEXITY OF SECURE DOMINATION PROBLEM IN GRAPHS

Haichao Wang [1], Yancai Zhao [2]

and

Yunping Deng [1]

[1] *Department of Mathematics*
Shanghai University of Electric Power
Shanghai 200090, China

[2] *Department of Basic Science*
Wuxi City College of Vocational Technology
Jiangsu 214153, China

e-mail: whchao2000@163.com
zhaoyc69@126.com
dyp612@163.com

Abstract

A dominating set of a graph G is a subset $D \subseteq V(G)$ such that every vertex not in D is adjacent to at least one vertex in D. A dominating set S of G is called a secure dominating set if each vertex $u \in V(G) \setminus S$ has one neighbor v in S such that $(S \setminus \{v\}) \cup \{u\}$ is a dominating set of G. The secure domination problem is to determine a minimum secure dominating set of G. In this paper, we first show that the decision version of the secure domination problem is NP-complete for star convex bipartite graphs and doubly chordal graphs. We also prove that the secure domination problem cannot be approximated within a factor of $(1-\varepsilon)\ln|V|$ for any $\varepsilon > 0$, unless NP$\subseteq$DTIME$\big(|V|^{O(\log\log|V|)}\big)$. Finally, we show that the secure domination problem is APX-complete for bounded degree graphs.

Keywords: secure domination, star convex bipartite graph, doubly chordal graph, NP-complete, APX-complete.

2010 Mathematics Subject Classification: 05C69.

References

[1] P. Alimonti and V. Kann, *Some APX-completeness results for cubic graphs*, Theoret. Comput. Sci. **237** (2000) 123–134.
doi:10.1016/S0304-3975(98)00158-3

[2] G. Ausiello, P. Crescenzi, G. Gambosi, V. Kann, A. Marchetti-Spaccamela and M. Protasi, Complexity and Approximation (Springer, Berlin, 1999).
doi:10.1007/978-3-642-58412-1

[3] A.A. Bertossi, *Dominating sets for split and bipartite graphs*, Inform. Process. Lett. **19** (1984) 37–40.
doi:10.1016/0020-0190(84)90126-1

[4] A. Brandstädt, F.F. Dragan, V. Chepoi and V. Voloshin, *Dually chordal graphs*, SIAM J. Discrete Math. **11** (1998) 437–455.
doi:10.1137/S0895480193253415

[5] A.P. Burger, M.A. Henning and J.H. van Vuuren, *Vertex covers and secure domination in graphs*, Quaest. Math. **31** (2008) 163–171.
doi:10.2989/QM.2008.31.2.5.477

[6] A.P. Burger, A.P. de Villiers and J.H. van Vuuren, *A linear algorithm for secure domination in trees*, Discrete Appl. Math. **171** (2014) 15–27.
doi:0.1016/j.dam.2014.02.001

[7] A.P. Burger, A.P. de Villiers and J.H. van Vuuren, *Edge criticality in secure graph domination*, Discrete Optim. **18** (2015) 111–122.
doi:10.1016/j.disopt.2015.08.001

[8] M. Chlebík and J. Chlebíková, *Approximation hardness of dominating set problems in bounded degree graphs*, Inform. and Comput. **206** (2008) 1264–1275.
doi:10.1016/j.ic.2008.07.003

[9] E.J. Cockayne, *Irredundance, secure domination and maximum degree in trees*, Discrete Math. **307** (2007) 12–17.
doi:10.1016/j.disc.2006.05.037

[10] E.J. Cockayne, P.J.P. Grobler, W.R. Gründlingh, J. Munganga and J.H. van Vuuren, *Protection of a graph*, Util. Math. **67** (2005) 19–32.

[11] D.R. Fulkerson and O.A. Gross, *Incidence matrices and interval graphs*, Pacific J. Math. **15** (1965) 835–855.
doi:10.2140/pjm.1965.15.835

[12] M.R. Garey and D.S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness (W.H. Freeman and Company, San Francisco, 1979).

[13] P.J.P. Grobler and C.M. Mynhardt, *Secure domination critical graphs*, Discrete Math. **309** (2009) 5820–5827.
doi:10.1016/j.disc.2008.05.050

[14] F. Harary and T.W. Haynes, *Double domination in graphs*, Ars Combin. **55** (2000) 201–213.

[15] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics (Marcel Dekker, New York, 1998).

[16] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs (Marcel Dekker, New York, 1998).

[17] W. Jiang, T. Liu, T. Ren and K. Xu, *Two hardness results on feedback vertex sets*, in: Proc. Fronties in Algorithmics and Algorytmic Aspects in Information and Managment, M. Atallah, X.Y. Li and B. Zhu (Ed(s)), Lecture Notes in Comput. Sci. **6681** (2011) 233–243.
doi:10.1007/978-3-642-21204-8_26

[18] R. Klasing and C. Laforest, *Hardness results and approximation algorithms of k-tuple domination in graphs*, Inform. Process. Lett. **89** (2004) 75–83.
doi:10.1016/j.ipl.2003.10.004

[19] W.F. Klostermeyer and C.M. Mynhardt, *Secure domination and secure total domination in graphs*, Discuss. Math. Graph Theory **28** (2008) 267–284.
doi:10.7151/dmgt.1405

[20] H.B. Merouane and M. Chellali, *On secure domination in graphs*, Inform. Process. Lett. **115** (2015) 786–790.
doi:10.1016/j.ipl.2015.05.006

[21] C.M. Mynhardt, H.C. Swart and L. Ungerer, *Excellent trees and secure domination*, Util. Math. **67** (2005) 255–267.

[22] C.H. Papadimitriou and M. Yannakakis, *Optimization, approximation and complexity classes*, J. Comput. System Sci. **43** (1991) 425–440.
doi:10.1016/0022-0000(91)90023-X

Received 26 April 2016
Revised 9 November 2016
Accepted 1 December 2016