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THE MINIMUM HARMONIC INDEX FOR UNICYCLIC GRAPHS WITH GIVEN DIAMETER

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Abstract

The harmonic index of a graph G is defined as the sum of the weights $\frac{2}{d(u)+d(v)}$ of all edges uv of G, where $d(u)$ denotes the degree of a vertex u in G. In this paper, we present the minimum harmonic index for unicyclic graphs with given diameter and characterize the corresponding extremal graphs. This answers an unsolved problem of Zhu and Chang [26].

Keywords: harmonic index, unicyclic graphs, diameter.

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