

## RAINBOW VERTEX-CONNECTION AND FORBIDDEN SUBGRAPHS

WENJING LI, XUELIANG LI<sup>1</sup>

AND

JINGSHU ZHANG

Center for Combinatorics and LPMC  
Nankai University  
Tianjin 300071, China

e-mail: liwenjing610@mail.nankai.edu.cn  
lxl@mail.nankai.edu.cn  
jszhang@mail.nankai.edu.cn

### Abstract

A path in a vertex-colored graph is called *vertex-rainbow* if its internal vertices have pairwise distinct colors. A vertex-colored graph  $G$  is *rainbow vertex-connected* if for any two distinct vertices of  $G$ , there is a vertex-rainbow path connecting them. For a connected graph  $G$ , the *rainbow vertex-connection number* of  $G$ , denoted by  $rvc(G)$ , is defined as the minimum number of colors that are required to make  $G$  rainbow vertex-connected. In this paper, we find all the families  $\mathcal{F}$  of connected graphs with  $|\mathcal{F}| \in \{1, 2\}$ , for which there is a constant  $k_{\mathcal{F}}$  such that, for every connected  $\mathcal{F}$ -free graph  $G$ ,  $rvc(G) \leq diam(G) + k_{\mathcal{F}}$ , where  $diam(G)$  is the diameter of  $G$ .

**Keywords:** vertex-rainbow path, rainbow vertex-connection, forbidden subgraphs.

**2010 Mathematics Subject Classification:** 05C15, 05C35, 05C38, 05C40.

### REFERENCES

- [1] G. Bacsó and Zs. Tuza, *Dominating cliques in  $P_5$ -free graphs*, Period. Math. Hungar. **21** (1990) 303–308.  
doi:10.1007/BF02352694

---

<sup>1</sup>Corresponding author.

- [2] J.A. Bondy and U.S.R. Murty, Graph Theory (GTM 244, Springer-Verlag, London, 2008).
- [3] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *Rainbow connection in graphs*, Math. Bohem. **133** (2008) 85–98.
- [4] L. Chen, X. Li and Y. Shi, *The complexity of determining the rainbow vertex-connection of a graph*, Theoret. Comput. Sci. **412** (2011) 4531–4535.  
doi:10.1016/j.tcs.2011.04.032
- [5] P. Holub, Z. Ryjáček, I. Schiermeyer and P. Vrána, *Rainbow connection and forbidden subgraphs*, Discrete Math. **338** (2015) 1706–1713.  
doi:10.1016/j.disc.2014.08.008
- [6] M. Krivelevich and R. Yuster, *The rainbow connection of a graph is (at most) reciprocal to its minimum degree*, J. Graph Theory **63** (2010) 185–191.  
doi:10.1002/jgt.20418
- [7] S. Li, X. Li and Y. Shi, *Note on the complexity of deciding the rainbow (vertex-)connectedness for bipartite graphs*, Appl. Math. Comput. **258** (2015) 155–161.  
doi:10.1016/j.amc.2015.02.015
- [8] X. Li and S. Liu, *Tight upper bound of the rainbow vertex-connection number for 2-connected graphs*, Discrete Appl. Math. **173** (2014) 62–69.  
doi:10.1016/j.dam.2014.04.002
- [9] X. Li and Y. Shi, *On the rainbow vertex-connection*, Discuss. Math. Graph Theory **33** (2013) 307–313.  
doi:10.7151/dmgt.1664
- [10] X. Li, Y. Shi and Y. Sun, *Rainbow connections of graphs: A survey*, Graphs Combin. **29** (2013) 1–38.  
doi:10.1007/s00373-012-1243-2
- [11] X. Li and Y. Sun, Rainbow Connections of Graphs (SpringerBriefs in Math., Springer-Verlag, New York, 2012).

Received 25 February 2016  
 Revised 21 October 2016  
 Accepted 21 October 2016