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UPPER BOUNDS FOR THE STRONG CHROMATIC INDEX OF HALIN GRAPHS

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Abstract

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The strong chromatic index of a graph G, denoted by $\chi'_s(G)$, is the minimum number of vertex induced matchings needed to partition the edge set of G. Let T be a tree without vertices of degree 2 and have at least one vertex of degree greater than 2. We construct a Halin graph G by drawing T on the plane and then drawing a cycle C connecting all its leaves in such a way that C forms the boundary of the unbounded face. We call T the characteristic tree of G. Let G denote a Halin graph with maximum degree Δ and characteristic tree T. We prove that $\chi'_s(G) \leqslant 2\Delta + 1$ when $\Delta \geqslant 4$. In addition, we show that if $\Delta = 4$ and G is not a wheel, then $\chi'_s(G) \leqslant \chi'_s(T) + 2$. A similar result for $\Delta = 3$ was established by Lih and Liu [21].

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