

ON INCIDENCE COLORING OF COMPLETE MULTIPARTITE AND SEMICUBIC BIPARTITE GRAPHS ¹

ROBERT JANCZEWSKI, MICHAŁ MAŁAFIEJSKI

*Department of Algorithms and System Modelling
Faculty of Electronics, Telecommunications and Informatics
Gdańsk University of Technology, Poland*

e-mail: skalar@eti.pg.gda.pl, michal@animima.org

AND

ANNA MAŁAFIEJSKA

*Department of Probability Theory and Biomathematics
Faculty of Physics and Applied Mathematics
Gdańsk University of Technology, Poland*

e-mail: anna@animima.org

Abstract

In the paper, we show that the incidence chromatic number χ_i of a complete k -partite graph is at most $\Delta+2$ (i.e., proving the *incidence coloring conjecture* for these graphs) and it is equal to $\Delta+1$ if and only if the smallest part has only one vertex (i.e., $\Delta = n-1$). Formally, for a complete k -partite graph $G = K_{r_1, r_2, \dots, r_k}$ with the size of the smallest part equal to $r_1 \geq 1$ we have

$$\chi_i(G) = \begin{cases} \Delta(G) + 1 & \text{if } r_1 = 1, \\ \Delta(G) + 2 & \text{if } r_1 > 1. \end{cases}$$

In the paper we prove that the incidence 4-coloring problem for semicubic bipartite graphs is $\mathcal{NP}$ -complete, thus we prove also the $\mathcal{NP}$ -completeness of $L(1, 1)$ -labeling problem for semicubic bipartite graphs. Moreover, we observe that the incidence 4-coloring problem is $\mathcal{NP}$ -complete for cubic graphs, which was proved in the paper [12] (in terms of generalized dominating sets).

Keywords: incidence coloring, complete multipartite graphs, semicubic graphs, subcubic graphs, $\mathcal{NP}$ -completeness, $L(1, 1)$ -labelling.

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