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A CHARACTERIZATION FOR 2-SELF-CENTERED GRAPHS

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Abstract

A graph is called 2-self-centered if its diameter and radius both equal to 2. In this paper, we begin characterizing these graphs by characterizing edge-maximal 2-self-centered graphs via their complements. Then we split characterizing edge-minimal 2-self-centered graphs into two cases. First, we characterize edge-minimal 2-self-centered graphs without triangles by introducing *specialized bi-independent covering* (SBIC) and a structure named *generalized complete bipartite graph* (GCBG). Then, we complete characterization by characterizing edge-minimal 2-self-centered graphs with some triangles. Hence, the main characterization is done since a graph is 2-self-centered if and only if it is a spanning subgraph of some edge-maximal 2-self-centered graphs and, at the same time, it is a spanning supergraph of some edge-minimal 2-self-centered graphs.

Keywords: self-centered graphs, specialized bi-independent covering (SBIC), generalized complete bipartite graphs (GCB).

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