EQUITABLE COLORINGS OF CORONA MULTIPRODUCTS OF GRAPHS

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Abstract

A graph is equitably k-colorable if its vertices can be partitioned into k independent sets in such a way that the numbers of vertices in any two sets differ by at most one. The smallest k for which such a coloring exists is known as the equitable chromatic number of G and denoted by $\chi_=(G)$. It is known that the problem of computation of $\chi_=(G)$ is NP-hard in general and remains so for corona graphs. In this paper we consider the same model of coloring in the case of corona multiproducts of graphs. In particular, we obtain some results regarding the equitable chromatic number for the l-corona product $G \circ^l H$, where G is an equitably 3- or 4-colorable graph and H is an r-partite graph, a cycle or a complete graph. Our proofs are mostly constructive in that they lead to polynomial algorithms for equitable coloring of such graph products provided that there is given an equitable coloring of G. Moreover, we confirm the Equitable Coloring Conjecture for

corona products of such graphs. This paper extends the results from [H. Furmańczyk, K. Kaliraj, M. Kubale and V.J. Vivin, *Equitable coloring of corona products of graphs*, Adv. Appl. Discrete Math. **11** (2013) 103–120].

Keywords: corona graph, equitable chromatic number, equitable coloring conjecture, equitable graph coloring, multiproduct of graphs, NP-completeness, polynomial algorithm.

2010 Mathematics Subject Classification: 05C15, 05C76.

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