Discussiones Mathematicae
Graph Theory 38 (2018) 97–106
doi:10.7151/dmgt.1991

CONSTANT SUM PARTITION OF SETS OF INTEGERS AND DISTANCE MAGIC GRAPHS

Sylwia Cichacz and Agnieszka Gőrlich

AGH University of Science and Technology
Faculty of Applied Mathematics

e-mail: cichacz@agh.edu.pl
forys@agh.edu.pl

Abstract

Let $A = \{1, 2, \ldots, tm+tn\}$. We shall say that A has the (m, n, t)-*balanced constant-sum-partition property* ((m, n, t)-*BCSP-property*) if there exists a partition of A into $2t$ pairwise disjoint subsets $A^1, A^2, \ldots, A^t, B^1, B^2, \ldots, B^t$ such that $|A^i| = m$ and $|B^i| = n$, and $\sum_{a \in A^i} a = \sum_{b \in B^j} b$ for $1 \leq i \leq t$ and $1 \leq j \leq t$. In this paper we give sufficient and necessary conditions for a set A to have the (m, n, t)-BCSP-property in the case when m and n are both even. We use this result to show some families of distance magic graphs.

Keywords: constant sum partition, distance magic labeling, product of graphs.

2010 Mathematics Subject Classification: 05C78.

References

[1] Y. Alavi, A.J. Boals, G. Chartrand, P. Erdős and O.R. Oellerman, *The ascending subgraph decomposition problem*, Congr. Numer. **58** (1987) 7–14.

[2] K. Ando, S. Gervacio and M. Kano, *Disjoint subsets of integers having a constant sum*, Discrete Math. **82** (1990) 7–11.
doi:10.1016/0012-365X(90)90040-O

[3] M. Anholcer and S. Cichacz, *Note on distance magic products $G \circ C_4$*, Graphs Combin. **31** (2015) 1117–1124.
doi:10.1007/s00373-014-1453-x

[4] M. Anholcer, S. Cichacz, I. Peterin and A. Tepeh, *Distance magic labeling and two products of graphs,* Graphs Combin. **31** (2015) 1125–1136.
doi:10.1007/s00373-014-1455-8

[5] S. Arumugam, D. Froncek and N. Kamatchi, *Distance magic graphs — A survey*, J. Indones. Math. Soc., Special Edition (2011) 11–26.

[6] S. Beena, *On Σ and Σ' labelled graphs*, Discrete Math. **309** (2009) 1783–1787.
doi:10.1016/j.disc.2008.02.038

[7] F.L. Chen, H.L. Fu, Y. Wang and J. Zhou, *Partition of a set of integers into subsets with prescribed sums*, Taiwanese J. Math. **9** (2005) 629–638.

[8] S. Cichacz, D. Froncek, E. Krop and C. Raridan, *Distance magic Cartesian products of graphs*, Discuss. Math. Graph Theory **36** (2016) 299–308.
doi:10.7151/dmgt.1852

[9] H. Enomoto and M. Kano, *Disjoint odd integer subsets having a constant even sum*, Discrete Math. **137** (1995) 189–193.
doi:10.1016/0012-365X(93)E0128-Q

[10] R.J. Faudree, A. Gyárfás and R.H. Schelp, *Graphs which have an ascending subgraph decomposition*, Congr. Numer. **59** (1987) 49–54.

[11] H.L. Fu and W.H. Hu, *A note on ascending subgraph decompositions of complete multipartite graphs*, Discrete Math. **226** (2001) 397–402.
doi:10.1016/S0012-365X(00)00171-0

[12] H.L. Fu and W.H. Hu, *A special partition of the set I_n*, Bull. Inst. Combin. Appl. **6** (1992) 57–61.

[13] H.L. Fu and W.H. Hu, *Ascending subgraph decompositions of regular graphs*, Discrete Math. **253** (2002) 11–18.
doi:10.1016/S0012-365X(01)00445-9

[14] H.L. Fu and W.H. Hu, *Disjoint odd integer subsets having a constant odd sum*, Discrete Math. **128** (1994) 143–150.
doi:10.1016/0012-365X(94)90108-2

[15] J.A. Gallian, *A dynamic survey of graph labeling*, Electron. J. Combin. (2016) #DS6.

[16] P. Gregor and P. Kovář, *Distance magic labelings of hypercubes*, Electron. Notes Discrete Math. **40** (2013) 145–149.
doi:10.1016/j.endm.2013.05.027

[17] T.R. Hagedorn, *Magic rectangles revisited*, Discrete Math. **207** (1999) 65–72.
doi:10.1016/S0012-365X(99)00041-2

[18] F. Harary, Graph Theory (Addison-Wesley, Reading, MA, 1994).

[19] T. Harmuth, *Über magische Quadrate und ähnliche Zahlenfiguren*, Arch. Math. Phys. **66** (1881) 286–313.

[20] T. Harmuth, *Über magische Rechtecke mit ungeraden Seitenzahlen*, Arch. Math. Phys. **66** (1881) 413–447.

[21] R. Hammack, W. Imrich and S. Klavžar, Handbook of Product Graphs, Second Edition (CRC Press, Boca Raton, FL, 2011).

[22] D. Kotlar, *Distance magic labeling in complete* 4*-partite graphs*, Graphs Combin. **32** (2016) 1027–1038.
doi:10.1007/s00373-015-1627-1

[23] A. Lladó and J. Moragas, *On the sumset partition problem*, Electron. Notes Discrete Math. **34** (2009) 15–19.
doi:10.1016/j.endm.2009.07.003

[24] A. Lladó and J. Moragas, *On the modular sumset partition problem*, European J. Combin. **33** (2012) 427–434.
doi:10.1016/j.ejc.2011.09.001

[25] K. Ma, H. Zhou and J. Zhou, *On the ascending star subgraph decomposition of star forests*, Combinatorica **14** (1994) 307–320.
doi:10.1007/BF01212979

[26] M. Miller, C. Rodger and R. Simanjuntak, *Distance magic labelings of graphs*, Australas. J. Combin. **28** (2003) 305–315.

[27] A. O'Neal and P.J. Slater, *Uniqueness of vertex magic constants*, SIAM J. Discrete Math. **27** (2013) 708–716.
doi:10.1137/110834421

[28] S.B. Rao, T. Singh and V. Prameswaran, *Some sigma labelled graphs* I, in: Graphs, Combinatorics, Algorithms and Applications, S. Arumugam, B.D. Acharya and S.B. Rao, (Eds.), (Narosa Publishing House, New Delhi, 2004) 125–133.

[29] V. Vilfred, Σ-Labelled Graphs and Circulant Graphs (Ph.D. Thesis, University of Kerala, Trivandrum, India, 1994).

Received 25 February 2016
Revised 7 October 2016
Accepted 8 October 2016