

CONSTANT SUM PARTITION OF SETS OF INTEGERS AND DISTANCE MAGIC GRAPHS

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Abstract

Let $A = \{1, 2, \dots, tm+tn\}$. We shall say that A has the (m, n, t) -balanced constant-sum-partition property ((m, n, t)-BCSP-property) if there exists a partition of A into $2t$ pairwise disjoint subsets $A^1, A^2, \dots, A^t, B^1, B^2, \dots, B^t$ such that $|A^i| = m$ and $|B^i| = n$, and $\sum_{a \in A^i} a = \sum_{b \in B^j} b$ for $1 \leq i \leq t$ and $1 \leq j \leq t$. In this paper we give sufficient and necessary conditions for a set A to have the (m, n, t) -BCSP-property in the case when m and n are both even. We use this result to show some families of distance magic graphs.

Keywords: constant sum partition, distance magic labeling, product of graphs.

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