

BOUNDS ON THE LOCATING ROMAN DOMINATION NUMBER IN TREES

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Abstract

A Roman dominating function (or just RDF) on a graph $G = (V, E)$ is a function $f : V \rightarrow \{0, 1, 2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$. The weight of an RDF f is the value $f(V(G)) = \sum_{u \in V(G)} f(u)$. An RDF f can be represented as $f = (V_0, V_1, V_2)$, where $V_i = \{v \in V : f(v) = i\}$ for $i = 0, 1, 2$. An RDF $f = (V_0, V_1, V_2)$ is called a locating Roman dominating function (or just LRDf) if $N(u) \cap V_2 \neq N(v) \cap V_2$ for any pair u, v of distinct vertices of V_0 . The locating Roman domination number $\gamma_R^L(G)$ is the minimum weight of an LRDf of G . In this paper, we study the locating Roman domination number in trees. We obtain lower and upper bounds for the locating Roman domination number of a tree in terms of its order and the number of leaves and support vertices, and characterize trees achieving equality for the bounds.

Keywords: Roman domination number, locating domination number, locating Roman domination number, tree.

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