

THE GRAPHS WHOSE PERMANENTAL POLYNOMIALS ARE SYMMETRIC

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Abstract

The permanental polynomial $\pi(G, x) = \sum_{i=0}^n b_i x^{n-i}$ of a graph G is symmetric if $b_i = b_{n-i}$ for each i . In this paper, we characterize the graphs with symmetric permanental polynomials. Firstly, we introduce the rooted product $H(K)$ of a graph H by a graph K , and provide a way to compute the permanental polynomial of the rooted product $H(K)$. Then we give a sufficient and necessary condition for the symmetric polynomial, and we prove that the permanental polynomial of a graph G is symmetric if and only if G is the rooted product of a graph by a path of length one.

Keywords: permanental polynomial, rooted product, matching.

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