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THE GRAPHS WHOSE PERMANENTAL POLYNOMIALS ARE SYMMETRIC

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Abstract

The permanental polynomial $\pi(G,x) = \sum_{i=0}^{n} b_i x^{n-i}$ of a graph G is symmetric if $b_i = b_{n-i}$ for each i. In this paper, we characterize the graphs with symmetric permanental polynomials. Firstly, we introduce the rooted product $H(K)$ of a graph H by a graph K, and provide a way to compute the permanental polynomial of the rooted product $H(K)$. Then we give a sufficient and necessary condition for the symmetric polynomial, and we prove that the permanental polynomial of a graph G is symmetric if and only if G is the rooted product of a graph by a path of length one.

Keywords: permanental polynomial, rooted product, matching.

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References

[1] M. Borowiecki and T. Jóźwiak, *Computing the permanental polynomial of a multigraph*, Discuss. Math. **5** (1982) 9–16.

[2] M. Borowiecki and T. Jóźwiak, *A note on characteristic and permanental polynomials of multigraphs*, in: Graph Theory, M. Borowiecki, J.W. Kenendy and M.M. Syslo (Ed(s)), (Springer-Verlag, Berlin, 1983) 75–78.
doi:10.1007/bfb0071615

[3] M. Borowiecki, *On spectrum and per-spectrum of graphs*, Publ. Inst. Math. (Beograd) (N.S.) **38** (1985) 31–33.

[4] Q. Chou, H. Liang and F. Bai, *Computing the permanental polynomial of the high level fullerene C_{70} with high precision*, MATCH Commun. Math. Comput. Chem. **73** (2015) 327–336.

[5] D.M. Cvetković, M. Doob and H. Sachs, Spectra of graphs (Academic Press, New York, 1980).

[6] I. Gutman and G.G. Cash, *Relations between the permanental and characteristic polynomials of fullerenes and benzenoid hydrocarbons*, MATCH Commun. Math. Comput. Chem. **45** (2002) 55–70.

[7] W. Li, *Graphs whose characteristic and permanental polynomials have coefficients of the same magnitude*, Discrete Math. **339** (2016) 2127–2135. doi:10.1016/j.disc.2016.02.019

[8] W. Li and H. Zhang, *On the permanental polynomials of matrices*, Bull. Malays. Math. Sci. Soc. **38** (2015) 1361–1374. doi:10.1007/s40840-014-0093-x

[9] W. Li and H. Zhang, *The permanental polynomials of certain graphs*, MATCH Commum. Math. Comput. Chem. **68** (2012) 871–888.

[10] L. Lovász and M.D. Plummer, Matching Theory (Elsevier, Amsterdam, 1986).

[11] R. Merris, K.R. Rebman and W. Watkins, *Permanental polynomials of graphs*, Linear Algebra Appl. **38** (1981) 273–288. doi:10.1016/0024-3795(81)90026-4

[12] Y. Shi, M. Dehmer, X. Li and I. Gutman, Graph Polynomials (CRC Press, Boca Raton, 2016).

[13] J. Turner, *Generalized marix functions and the graph isomorphism problem*, SIAM J. Appl. Math. **16** (1968) 520–526. doi:10.1137/0116041

[14] T. Wu and H. Zhang, *Per-spectral and adjacency spectral characterizations of a complete graph removing six edges*, Discrete Appl. Math. **203** (2016) 158–170. doi:10.1016/j.dam.2015.09.014

[15] W. Yan and F. Zhang, *On the permanental polynomials of some graphs*, J. Math. Chem. **35** (2004) 175–188. doi:10.1023/B:JOMC.0000033254.54822.f8

[16] H. Zhang and W. Li, *Computing the permanental polynomials of bipartite graphs by Pfaffian orientation*, Discrete Appl. Math. **160** (2012) 2069–2074. doi:10.1016/j.dam.2012.04.007

[17] H. Zhang, S. Liu and W. Li, *A note on the permanental roots of bipartite graphs*, Discuss. Math. Graph Theory **34** (2014) 49–56. doi:10.7151/dmgt.1704

[18] H. Zhang, T. Wu and H. Lai, *Per-spectral characterizations of some edge-deleted subgraphs of a complete graph*, Linear Multilinear Algebra **63** (2015) 397–410. doi:10.1080/03081087.2013.869592

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