

A SHARP LOWER BOUND FOR THE GENERALIZED 3-EDGE-CONNECTIVITY OF STRONG PRODUCT GRAPHS

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Abstract

The generalized k -connectivity $\kappa_k(G)$ of a graph G , mentioned by Hager in 1985, is a natural generalization of the path-version of the classical connectivity. As a natural counterpart of this concept, Li *et al.* in 2011 introduced the concept of generalized k -edge-connectivity which is defined as $\lambda_k(G) = \min\{\lambda_G(S) \mid S \subseteq V(G) \text{ and } |S| = k\}$, where $\lambda_G(S)$ denote the maximum number ℓ of pairwise edge-disjoint trees $T_1, T_2, \dots, T_\ell$ in G such that $S \subseteq V(T_i)$ for $1 \leq i \leq \ell$. In this paper we get a sharp lower bound for the generalized 3-edge-connectivity of the strong product of any two connected graphs.

Keywords: generalized connectivity, generalized edge-connectivity, strong product.

2010 Mathematics Subject Classification: 05C05, 05C40, 05C76.

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Received 13 February 2016

Revised 17 August 2016

Accepted 17 August 2016