

PER-SPECTRAL CHARACTERIZATIONS OF SOME BIPARTITE GRAPHS

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Abstract

A graph is said to be characterized by its permanental spectrum if there is no other non-isomorphic graph with the same permanental spectrum. In this paper, we investigate when a complete bipartite graph $K_{p,p}$ with some edges deleted is determined by its permanental spectrum. We first prove that a graph obtained from $K_{p,p}$ by deleting all edges of a star $K_{1,l}$, provided $l < p$, is determined by its permanental spectrum. Furthermore, we show that all graphs with a perfect matching obtained from $K_{p,p}$ by removing five or fewer edges are determined by their permanental spectra.

Keywords: permanent, permanental polynomial, per-spectrum, cospectral.

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