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THE EXISTENCE OF $P_{\geq 3}$-FACTOR COVERED GRAPHS

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Abstract

A spanning subgraph F of a graph G is called a $P_{\geq 3}$-factor of G if every component of F is a path of order at least 3. A graph G is called a $P_{\geq 3}$-factor covered graph if G has a $P_{\geq 3}$-factor including e for any $e \in E(G)$. In this paper, we obtain three sufficient conditions for graphs to be $P_{\geq 3}$-factor covered graphs. Furthermore, it is shown that the results are sharp.

Keywords: $P_{\geq 3}$-factor, $P_{\geq 3}$-factor covered graph, toughness, isolated toughness, regular graph.

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