

CONSTANT 2-LABELLINGS AND AN APPLICATION TO (r, a, b) -COVERING CODES

SYLVAIN GRAVIER

CNRS, Institut Fourier, Grenoble, France

e-mail: sylvain.gravier@ujf-grenoble.fr

AND

ÉLISE VANDOMME

University of Liège, Liège, Belgium

e-mail: e.vandomme@ulg.ac.be

Abstract

We introduce the concept of constant 2-labelling of a vertex-weighted graph and show how it can be used to obtain perfect weighted coverings. Roughly speaking, a constant 2-labelling of a vertex-weighted graph is a black and white colouring of its vertex set which preserves the sum of the weights of black vertices under some automorphisms. We study constant 2-labellings on four types of vertex-weighted cycles. Our results on cycles allow us to determine (r, a, b) -codes in \mathbb{Z}^2 whenever $|a - b| > 4$, $r \geq 2$ and we give the precise values of a and b . This is a refinement of Axenovich's theorem proved in 2003.

Keywords: covering codes, weighted codes, infinite grid, vertex-weighted graphs.

2010 Mathematics Subject Classification: 05C63, 05C70, 05C78.

REFERENCES

- [1] M. Axenovich, *On multiple coverings of the infinite rectangular grid with balls of constant radius*, *Discrete Math.* **268** (2003) 31–48.
doi:10.1016/S0012-365X(02)00744-6
- [2] N. Biggs, *Perfect codes in graphs*, *J. Combin. Theory Ser. B* **15** (1973) 289–296.
doi:10.1016/0095-8956(73)90042-7
- [3] G. Cohen, I. Honkala, S. Litsyn and A. Lobstein, *Covering Codes* (North-Holland Publishing Co., Amsterdam, 1997).

- [4] G. Cohen, I. Honkala, S. Litsyn and H. Mattson, Jr, *Weighted coverings and packings*, IEEE Trans. Inform. Theory **41** (1995) 1856–1867.
doi:10.1109/18.476311
- [5] P. Dorbec, S. Gravier, I. Honkala and M. Mollard, *Weighted codes in Lee metrics*, Des. Codes Cryptogr. **52** (2009) 209–218.
doi:10.1007/s10623-009-9277-z
- [6] C.D. Godsil, Algebraic Combinatorics (Chapman & Hall, New York, 1993).
- [7] S.W. Golomb and L.R. Welch, *Algebraic coding and the Lee metric*, in: Error Correcting Codes (Proc. Sympos. Math. Res. Center, Madison, Wis., 1968), (John Wiley, New York, 1968) 175–194.
doi:10.1137/0118025
- [8] S.W. Golomb and L.R. Welch, *Perfect codes in the Lee metric and the packing of polyominoes*, SIAM J. Appl. Math. **18** (1970) 302–317.
- [9] S. Gravier, A. Lacroix and S. Slimani, *(a, b)-codes in $\mathbb{Z}/n\mathbb{Z}$* , Discrete Appl. Math. **161** (2013) 612–617.
- [10] S. Gravier, M. Mollard and C. Payan, *Variations on tilings in the Manhattan metric*, Geom. Dedicata **76** (1999) 265–273.
doi:10.1023/A:1005106901394
- [11] P. Horak, *On perfect Lee codes*, Discrete Math. **309** (2009) 5551–5561.
doi:10.1016/j.disc.2008.03.019
- [12] J. Kratochvíl, *Perfect codes in general graphs*, in: Combinatorics (Eger, 1987), Colloq. Math. Soc. János Bolyai, North-Holland, Amsterdam **52** (1988) 357–364.
- [13] S. Puzynina, *Periodicity of perfect colorings of an infinite rectangular grid*, Diskretn. Anal. Issled. Oper. Ser. 1 **11** (2004) 79–92.
- [14] S. Puzynina, *On periodicity of generalized two-dimensional infinite words*, Inform. and Comput. **207** (2009) 1315–1328.
doi:10.1016/j.ic.2009.03.005
- [15] J.A. Telle, *Complexity of domination-type problems in graphs*, Nordic J. Comput. **1** (1994) 157–171.
- [16] É. Vandomme, Contributions to Combinatorics on Words in an Abelian Context and Covering Problems in Graphs (PhD Thesis, University of Grenoble and University of Liege, 2015).
hdl.handle.net/2268/176364

Received 16 November 2015

Revised 11 July 2016

Accepted 11 July 2016