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THE SIGNED TOTAL ROMAN k-DOMATIC NUMBER OF A GRAPH

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Abstract

Let $k \geq 1$ be an integer. A *signed total Roman k-dominating function* on a graph G is a function $f : V(G) \longrightarrow \{-1, 1, 2\}$ such that $\sum_{u \in N(v)} f(u) \geq k$ for every $v \in V(G)$, where $N(v)$ is the neighborhood of v, and every vertex $u \in V(G)$ for which $f(u) = -1$ is adjacent to at least one vertex w for which $f(w) = 2$. A set $\{f_1, f_2, \ldots, f_d\}$ of distinct signed total Roman k-dominating functions on G with the property that $\sum_{i=1}^{d} f_i(v) \leq k$ for each $v \in V(G)$, is called a *signed total Roman k-dominating family* (of functions) on G. The maximum number of functions in a signed total Roman k-dominating family on G is the *signed total Roman k-domatic number* of G, denoted by $d_{stR}^k(G)$. In this paper we initiate the study of signed total Roman k-domatic numbers in graphs, and we present sharp bounds for $d_{stR}^k(G)$. In particular, we derive some Nordhaus-Gaddum type inequalities. In addition, we determine the signed total Roman k-domatic number of some graphs.

Keywords: signed total Roman k-dominating function, signed total Roman k-domination number, signed total Roman k-domatic number.

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