

THE DEGREE-DIAMETER PROBLEM FOR OUTERPLANAR GRAPHS

PETER DANKELMANN

Department of Mathematics
University of Johannesburg
Johannesburg, South Africa

e-mail: pdankelmann@uj.ac.za

ELIZABETH JONCK

School of Mathematics
University of the Witwatersrand
Johannesburg, South Africa

e-mail: Betsie.Jonck@wits.ac.za

AND

TOMÁŠ VETRÍK

Department of Mathematics and Applied Mathematics
University of the Free State
Bloemfontein, South Africa

e-mail: vetrikt@ufs.ac.za

Abstract

For positive integers Δ and D we define $n_{\Delta,D}$ to be the largest number of vertices in an outerplanar graph of given maximum degree Δ and diameter D . We prove that $n_{\Delta,D} = \Delta^{\frac{D}{2}} + O\left(\Delta^{\frac{D}{2}-1}\right)$ if D is even, and $n_{\Delta,D} = 3\Delta^{\frac{D-1}{2}} + O\left(\Delta^{\frac{D-1}{2}-1}\right)$ if D is odd. We then extend our result to maximal outerplanar graphs by showing that the maximum number of vertices in a maximal outerplanar graph of maximum degree Δ and diameter D asymptotically equals $n_{\Delta,D}$.

Keywords: outerplanar, diameter, degree, degree-diameter problem, distance, separator theorem.

2010 Mathematics Subject Classification: 05C35, 05C12.

REFERENCES

- [1] M. Abas, *Cayley graphs of diameter two and any degree with order half of the Moore bound*, Discrete Appl. Math. **173** (2014) 1–7.
doi:10.1016/j.dam.2014.04.005
- [2] C. Balbuena, M. Miller, J. Širáň and M. Ždimalová, *Large vertex-transitive graphs of diameter 2 from incidence graphs of biaffine planes*, Discrete Math. **313** (2013) 2014–2019.
doi:10.1016/j.disc.2013.03.007
- [3] C. Dalfó, C. Huemer and J. Salas, *The degree/diameter problem in maximal planar bipartite graphs*, Electron. J. Combin. **23** (1) (2016) #P60.
- [4] P. Dankelmann, D. Erwin, W.D. Goddard, S. Mukwembi and H.C. Swart, *A characterisation of eccentric sequences of maximal outerplanar graphs*, Australas. J. Combin. **58** (2014) 376–391.
- [5] P. Dankelmann and T. Vetrík, *The degree-diameter problem for claw-free graphs and hypergraphs*, J. Graph Theory **75** (2014) 105–123.
doi:10.1002/jgt.21716
- [6] A.M. Farley and A. Proskurowski, *Computation of the center and diameter of outerplanar graphs*, Discrete Appl. Math. **2** (1980) 185–191.
doi:10.1016/0166-218X(80)90039-6
- [7] M. Fellows, P. Hell and K. Seyffarth, *Large planar graphs with given diameter and maximum degree*, Discrete Appl. Math **61** (1995) 133–153.
doi:10.1016/0166-218X(94)00011-2
- [8] R. Fera-Purón and G. Pineda-Villavicencio, *On bipartite graphs of defect at most 4*, Discrete Appl. Math **160** (2012) 140–154.
doi:10.1016/j.dam.2011.09.002
- [9] P. Hell and K. Seyffarth, *Largest planar graphs of diameter two and fixed maximum degree*, Discrete Math. **111** (1993) 313–322.
doi:10.1016/0012-365X(93)90166-Q
- [10] R.J. Lipton and R.E. Tarjan, *A separator theorem for planar graphs*, SIAM J. Appl. Math. **36** (1979) 177–189.
doi:10.1137/0136016
- [11] E. Nevo, G. Pineda-Villavicencio and D.R. Wood, *On the maximum order of graphs embedded in surfaces*, J. Combin. Theory Ser. B **119** (2016) 28–41.
doi:10.1016/j.jctb.2015.12.004
- [12] A. Proskurowski, *Centers of maximal outerplanar graphs*, J. Graph Theory **4** (1980) 75–79.
doi:10.1002/jgt.3190040108
- [13] K. Seyffarth, *Maximal planar graphs of diameter two*, J. Graph Theory **13** (1989) 619–648.
doi:10.1002/jgt.3190130512

- [14] S.A. Tishchenko, *Maximum size of a planar graph with given degree and even diameter*, European J. Combin. **33** (2012) 380–396.
doi:10.1016/j.ejc.2011.09.005
- [15] T. Vetrík, *Abelian Cayley graphs of given degree and diameter 2 and 3*, Graphs Combin. **30** (2014) 1587–1591.
doi:10.1007/s00373-013-1361-5
- [16] T. Vetrík, R. Simanjuntak and E. Baskoro, *Large bipartite Cayley graphs of given degree and diameter*, Discrete Math. **311** (2011) 324–326.
doi:10.1016/j.disc.2010.10.015
- [17] Y. Yang, J. Lin and Y. Dai, *Largest planar graphs and largest maximal planar graphs of diameter two*, J. Comput. Appl. Math. **144** (2002) 349–358.
doi:10.1016/S0377-0427(01)00572-6

Received 8 January 2016

Revised 5 August 2016

Accepted 5 August 2016