

## HEAVY SUBGRAPHS, STABILITY AND HAMILTONICITY

BINLONG LI<sup>a,c</sup> AND BO NING<sup>1b</sup>

<sup>a</sup>*Department of Applied Mathematics  
Northwestern Polytechnical University  
Xi'an, Shaanxi 710072, P.R. China*

<sup>b</sup>*Center for Applied Mathematics  
Tianjin University  
Tianjin 300072, P.R. China*

<sup>c</sup>*European Centre of Excellence NTIS  
306 14 Pilsen, Czech Republic*

e-mail: bo.ning@tju.edu.cn

### Abstract

Let  $G$  be a graph. Adopting the terminology of Broersma *et al.* and Čada, respectively, we say that  $G$  is 2-heavy if every induced claw ( $K_{1,3}$ ) of  $G$  contains two end-vertices each one has degree at least  $|V(G)|/2$ ; and  $G$  is o-heavy if every induced claw of  $G$  contains two end-vertices with degree sum at least  $|V(G)|$  in  $G$ . In this paper, we introduce a new concept, and say that  $G$  is  $S$ -c-heavy if for a given graph  $S$  and every induced subgraph  $G'$  of  $G$  isomorphic to  $S$  and every maximal clique  $C$  of  $G'$ , every non-trivial component of  $G' - C$  contains a vertex of degree at least  $|V(G)|/2$  in  $G$ . Our original motivation is a theorem of Hu from 1999 that can be stated, in terms of this concept, as every 2-connected 2-heavy and  $N$ -c-heavy graph is hamiltonian, where  $N$  is the graph obtained from a triangle by adding three disjoint pendant edges. In this paper, we will characterize all connected graphs  $S$  such that every 2-connected o-heavy and  $S$ -c-heavy graph is hamiltonian. Our work results in a different proof of a stronger version of Hu's theorem. Furthermore, our main result improves or extends several previous results.

**Keywords:** heavy subgraphs, hamiltonian graphs, closure theory.

**2010 Mathematics Subject Classification:** 05C38, 05C45.

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<sup>1</sup>Corresponding author (Bo Ning).

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Received 22 June 2015  
 Revised 5 February 2016  
 Accepted 11 June 2016