A DEGREE CONDITION IMPLYING ORE-TYPE CONDITION FOR EVEN [2, b]-FACTORS IN GRAPHS

SHOICHI TSUCHIYA

School of Network and Information
Senshu University, 2-1-1 Higashimita, Tama-ku
Kawasaki-shi, Kanagawa 214-8580, Japan

AND

TAKAMASA YASHIMA

Department of Mathematical Information Science
Tokyo University of Science, 1-3 Kagurazaka
Shinjuku-ku, Tokyo 162-8601, Japan

e-mail: takamasa.yashima@gmail.com

Abstract

For a graph $G$ and even integers $b \geq a \geq 2$, a spanning subgraph $F$ of $G$ such that $a \leq \deg_F(x) \leq b$ and $\deg_F(x)$ is even for all $x \in V(F)$ is called an even $[a, b]$-factor of $G$. In this paper, we show that a 2-edge-connected graph $G$ of order $n$ has an even $[2, b]$-factor if $\max\{\deg_G(x), \deg_G(y)\} \geq \max\{\frac{2n}{a+1}, 3\}$ for any nonadjacent vertices $x$ and $y$ of $G$. Moreover, we show that for $b \geq 3a$ and $a \geq 2$, there exists an infinite family of 2-edge-connected graphs $G$ of order $n$ with $\delta(G) \geq a$ such that $G$ satisfies the condition $\deg_G(x) + \deg_G(y) > \frac{2n}{a+1}$ for any nonadjacent vertices $x$ and $y$ of $G$, but has no even $[a, b]$-factors. In particular, the infinite family of graphs gives a counterexample to the conjecture of Matsuda on the existence of an even $[a, b]$-factor.

Keywords: $[a, b]$-factor, even factor, 2-edge-connected, minimum degree.

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References


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