A DEGREE CONDITION IMPLYING ORE-TYPE CONDITION FOR EVEN \([2, b]\)-FACTORS IN GRAPHS

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Abstract

For a graph \(G\) and even integers \(b \geq a \geq 2\), a spanning subgraph \(F\) of \(G\) such that \(a \leq \deg_F(x) \leq b\) and \(\deg_F(x)\) is even for all \(x \in V(F)\) is called an even \([a, b]\)-factor of \(G\). In this paper, we show that a 2-edge-connected graph \(G\) of order \(n\) has an even \([2, b]\)-factor if \(\max\{\deg_G(x), \deg_G(y)\} \geq \max\{\frac{2n}{2+a}, 3\}\) for any nonadjacent vertices \(x\) and \(y\) of \(G\). Moreover, we show that for \(b \geq 3a\) and \(a > 2\), there exists an infinite family of 2-edge-connected graphs \(G\) of order \(n\) with \(\delta(G) \geq a\) such that \(G\) satisfies the condition \(\deg_G(x) + \deg_G(y) > \frac{2an}{a+3}\) for any nonadjacent vertices \(x\) and \(y\) of \(G\), but has no even \([a, b]\)-factors. In particular, the infinite family of graphs gives a counterexample to the conjecture of Matsuda on the existence of an even \([a, b]\)-factor.

Keywords: \([a, b]\)-factor, even factor, 2-edge-connected, minimum degree.

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References


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