# A DEGREE CONDITION IMPLYING ORE-TYPE CONDITION FOR EVEN [ $2, b]$-FACTORS IN GRAPHS 

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#### Abstract

For a graph $G$ and even integers $b \geqslant a \geqslant 2$, a spanning subgraph $F$ of $G$ such that $a \leqslant \operatorname{deg}_{F}(x) \leqslant b$ and $\operatorname{deg}_{F}(x)$ is even for all $x \in V(F)$ is called an even $[a, b]$-factor of $G$. In this paper, we show that a 2 -edge-connected graph $G$ of order $n$ has an even $[2, b]$-factor if $\max \left\{\operatorname{deg}_{G}(x), \operatorname{deg}_{G}(y)\right\} \geqslant$ $\max \left\{\frac{2 n}{2+b}, 3\right\}$ for any nonadjacent vertices $x$ and $y$ of $G$. Moreover, we show that for $b \geqslant 3 a$ and $a>2$, there exists an infinite family of 2-edge-connected graphs $G$ of order $n$ with $\delta(G) \geqslant a$ such that $G$ satisfies the condition $\operatorname{deg}_{G}(x)+\operatorname{deg}_{G}(y)>\frac{2 a n}{a+b}$ for any nonadjacent vertices $x$ and $y$ of $G$, but has no even $[a, b]$-factors. In particular, the infinite family of graphs gives a counterexample to the conjecture of Matsuda on the existence of an even [ $a, b]$-factor.


Keywords: $[a, b]$-factor, even factor, 2-edge-connected, minimum degree.
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