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A DEGREE CONDITION IMPLYING ORE-TYPE CONDITION FOR EVEN $[2, b]$-FACTORS IN GRAPHS

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Abstract

For a graph G and even integers $b \geqslant a \geqslant 2$, a spanning subgraph F of G such that $a \leqslant \deg_F(x) \leqslant b$ and $\deg_F(x)$ is even for all $x \in V(F)$ is called an even $[a, b]$-factor of G. In this paper, we show that a 2-edge-connected graph G of order n has an even $[2, b]$-factor if $\max\{\deg_G(x), \deg_G(y)\} \geqslant \max\{\frac{2n}{2+b}, 3\}$ for any nonadjacent vertices x and y of G. Moreover, we show that for $b \geqslant 3a$ and $a > 2$, there exists an infinite family of 2-edge-connected graphs G of order n with $\delta(G) \geqslant a$ such that G satisfies the condition $\deg_G(x) + \deg_G(y) > \frac{2an}{a+b}$ for any nonadjacent vertices x and y of G, but has no even $[a, b]$-factors. In particular, the infinite family of graphs gives a counterexample to the conjecture of Matsuda on the existence of an even $[a, b]$-factor.

Keywords: $[a, b]$-factor, even factor, 2-edge-connected, minimum degree.

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References

[1] M. Kouider and P.D. Vestergaard, *On even* $[2, b]$*-factors in graphs*, Australas. J. Combin. **27** (2003) 139–147.

[2] M. Kouider and P.D. Vestergaard, *Even $[a, b]$-factors in graphs*, Discuss. Math. Graph Theory **24** (2004) 431–441.
doi:10.7151/dmgt.1242

[3] L. Lovász, *Subgraphs with prescribed valencies*, J. Combin. Theory **8** (1970) 391–416.
doi:10.1016/S0021-9800(70)80033-3

[4] H. Matsuda, *Ore-type conditions for the existence of even $[2, b]$-factors in graphs*, Discrete Math. **304** (2005) 51–61.
doi:10.1016/j.disc.2005.09.009

[5] W.T. Tutte, *Graph factors*, Combinatorica **1** (1981) 79–97.
doi:10.1007/BF02579180

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