Discussiones Mathematicae
Graph Theory 37 (2017) 573–586
doi:10.7151/dmgt.1959

REGULARITY AND PLANARITY OF TOKEN GRAPHS

Walter Carballosa

Department of Mathematics and Statistics
Florida International University, Miami, FL

e-mail: waltercarb@gmail.com

Ruy Fabila-Monroy

Departamento de Matemáticas
CINVESTAV, Mexico

e-mail: ruyfabila@math.cinvestav.edu.mx

Jesús Leaños

Faculty of Natural Sciences and Mathematics
University of Maribor, Slovenia

e-mail: jesus.leanos@gmail.com

and

Luis Manuel Rivera

Unidad Académica de Matemáticas
Universidad Autónoma de Zacatecas, Zac., Mexico

e-mail: luismanuel.rivera@gmail.com

Abstract

Let $G = (V, E)$ be a graph of order n and let $1 \leq k < n$ be an integer. The *k-token graph* of G is the graph whose vertices are all the k-subsets of V, two of which are adjacent whenever their symmetric difference is a pair of adjacent vertices in G. In this paper we characterize precisely, for each value of k, which graphs have a regular k-token graph and which connected graphs have a planar k-token graph.

Keywords: token graph, Johnson graph, regularity, planarity.

2010 Mathematics Subject Classification: 05C10, 05C69.

References

[1] J.M. Boyer and J.W. Myrvold, *On the cutting edge: simplified $O(n)$ planarity by edge addition*, J. Graph Algorithms Appl. **8** (2004) 241–273. doi:10.7155/jgaa.00091

[2] M. Behzad and S.E. Mahmoodian, *On topological invariants of the product of graphs*, Canad. Math. Bull. **12** (1969) 175–166. doi:10.4153/CMB-1969-017-3

[3] T. Etzion and S. Bitan, *On the chromatic number, colorings, and codes of the Johnson graph*, Discrete Appl. Math. **70** (1996) 163–175. doi:10.1016/0166-218X(96)00104-7

[4] R. Fabila-Monroy, D. Flores-Peñaloza, C. Huemer, F. Hurtado, J. Urrutia and D.R. Wood, *Token graphs*, Graphs Combin. **28** (2012) 365–380. doi:10.1007/s00373-011-1055-9

[5] R. Graham and N. Sloane, *Lower bounds for constant weight codes*, IEEE Trans. Inform. Theory **26** (1980) 37–43. doi:10.1109/TIT.1980.1056141

[6] J. Guo, K. Wang and F. Li, *Metric dimension of some distance-regular graphs*, J. Comb. Optim. **26** (2013) 190–197. doi:10.1007/s10878-012-9459-x

[7] F. Harary, Graph Theory (Addison-Wesley, Reading, MA, 1969).

[8] K. Kuratowski, *Sur le problème des courbes gauches en topologie*, Fund. Math. **15** (1930) 271–283.

[9] R.A. Liebler and C.E. Praeger, *Neighbour-transitive codes in Johnson graphs*, Des. Codes Cryptogr. **73** (2014) 1–25. doi:10.1007/s10623-014-9982-0

[10] B.D. McKay and A. Piperno, *Practical graph isomorphism* II, J. Symbolic Comput. **60** (2013) 94–112. doi:10.1016/j.jsc.2013.09.003

[11] M. Neunhöffer, and C.E. Praeger, *Sporadic neighbour-transitive codes in Johnson graphs*, Des. Codes Cryptogr. **72** (2014) 141–152. doi:10.1007/s10623-013-9853-0

[12] W.A. Stein, *et al.*, Sage Mathematics Software (Version 6.8), The Sage Development Team, 2015. http://www.sagemath.org.

[13] K. Wagner, *Über eine Eigenschaft der ebenen Komplexe*, Math. Ann. **114** (1937) 570–590. doi:10.1007/BF01594196

Received 14 December 2015
Revised 26 May 2016
Accepted 26 May 2016