

FORBIDDEN STRUCTURES FOR PLANAR PERFECT CONSECUTIVELY COLOURABLE GRAPHS

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Abstract

A *consecutive colouring* of a graph is a proper edge colouring with positive integers in which the colours of edges incident with each vertex form an interval of integers. The idea of this colouring was introduced in 1987 by Asratian and Kamalian under the name of *interval colouring*. Sevastjanov showed that the corresponding decision problem is *NP*-complete even restricted to the class of bipartite graphs. We focus our attention on the class of consecutively colourable graphs whose all induced subgraphs are consecutively colourable, too. We call elements of this class *perfect consecutively colourable* to emphasise the conceptual similarity to perfect graphs. Obviously, the class of perfect consecutively colourable graphs is induced hereditary, so it can be characterized by the family of induced forbidden graphs. In this work we give a necessary and sufficient conditions that must be satisfied by the generalized Sevastjanov rosette to be an induced forbidden graph for the class of perfect consecutively colourable graphs. Along the way, we show the exact values of the deficiency of all generalized Sevastjanov rosettes, which improves the earlier known estimating result. It should be mentioned that the deficiency of a graph measures its closeness to the class of consecutively colourable graphs. We motivate the investigation of graphs considered here by showing their connection to the class of planar perfect consecutively colourable graphs.

Keywords: edge colouring, consecutive (interval) colouring, deficiency, Sevastjanov graph, forbidden graph.

2010 Mathematics Subject Classification: 05C75, 05C15, 05C35.

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Received 19 April 2016
 Revised 1 October 2016
 Accepted 1 October 2016