

PRIME FACTORIZATION AND DOMINATION IN THE HIERARCHICAL PRODUCT OF GRAPHS

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Abstract

In 2009, Barrière, Dalfó, Fiol, and Mitjana introduced the generalized hierarchical product of graphs. This operation is a generalization of the Cartesian product of graphs. It is known that every connected graph has a unique prime factor decomposition with respect to the Cartesian product. We generalize this result to show that connected graphs indeed have a unique prime factor decomposition with respect to the generalized hierarchical product. We also give preliminary results on the domination number of generalized hierarchical products.

Keywords: generalized hierarchical product, Cartesian product, prime factor decomposition.

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REFERENCES

- [1] M. Azari and A. Iranmanesh, *Chemical graphs constructed from rooted products and their Zagreb indices*, MATCH Commun. Math. Comput. Chem. **70** (2013) 901–919.
- [2] L. Barrière, F. Comellas, C. Dalfó and M.A. Fiol, *The hierarchical product of graphs*, Discrete Appl. Math. **157** (2009) 36–48.
doi:10.1016/j.dam.2008.04.018

- [3] L. Barrière, C. Dalfó, M.A. Fiol and M. Mitjana, *The generalized hierarchical product of graphs*, Discrete Math. **309** (2009) 3871–3881.
doi:10.1016/j.disc.2008.10.028
- [4] B. Brešar, P. Dorbec, W. Goddard, B. Hartnell, M.A. Henning, S. Klavžar and D. Rall, *Vizing’s conjecture: A survey and recent results*, J. Graph Theory **69** (2012) 46–76.
doi:10.1002/jgt.20565
- [5] J.F. Fink, M.S. Jacobson, L.F. Kinch and J. Roberts, *On graphs having domination number half their order*, Period. Math. Hungar. **16** (1985) 287–293.
doi:10.1007/BF01848079
- [6] C.D. Godsil and B.D. McKay, *A new graph product and its spectrum*, Bull. Aust. Math. Soc. **18** (1978) 21–28.
doi:10.1017/S0004972700007760
- [7] R. Hammack, W. Imrich and S. Klavžar, *Handbook of Product Graphs*, Second Edition (CRC Press, Boca Raton, FL, 2011).
- [8] M.S. Jacobson and L.F. Kinch, *On the domination of the products of graphs II: Trees*, J. Graph Theory **10** (1986) 97–106.
doi:10.1002/jgt.3190100112
- [9] S. Jung, S. Kim and B. Kahng, *Geometric fractal growth model for scale-free networks*, Phys. Rev. E **65** (2002) 056101.
doi:10.1103/PhysRevE.65.056101
- [10] K.M. Koh, D.G. Rogers and T. Tran, *Products of graceful trees*, Discrete Math. **31** (1980) 279–292.
doi:10.1016/0012-365X(80)90139-9
- [11] D. Kuziak, I.G. Yero and J.A. Rodriguez-Velazquez, *Strong metric dimension of rooted product graphs*, Int. J. Comput. Math. **93** (2016) 1265–1280.
doi:10.1080/00207160.2015.1061656
- [12] A. Meir and J.W. Moon, *Relations between packing and covering numbers of a tree*, Pacific J. Math. **61** (1975) 225–233.
doi:10.2140/pjm.1975.61.225
- [13] M.E.J. Newman, *The structure and function of complex networks*, SIAM Rev. **45** (2003) 167–256.
doi:10.1137/S003614450342480
- [14] G. Sabidussi, *Graph multiplication*, Math. Z. **72** (1960) 446–457.
doi:10.1007/BF01162967
- [15] V.G. Vizing, *The cartesian product of graphs*, Vyčisl. Sistemy **9** (1963) 30–43, in Russian. English translation in: Comp. El. Syst. **2** (1966) 352–365.

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