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KALEIDOSCOPIC COLORINGS OF GRAPHS

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Abstract

For an r-regular graph G, let $c : E(G) \to [k] = \{1, 2, \ldots, k\}$, $k \geq 3$, be an edge coloring of G, where every vertex of G is incident with at least one edge of each color. For a vertex v of G, the multiset-color $c_m(v)$ of v is defined as the ordered k-tuple $(a_1, a_2, \ldots, a_k)$ or $a_1 a_2 \cdots a_k$, where a_i $(1 \leq i \leq k)$ is the number of edges in G colored i that are incident with v. The edge coloring c is called k-kaleidoscopic if $c_m(u) \neq c_m(v)$ for every two distinct vertices u and v of G. A regular graph G is called a k-kaleidoscope if G has a k-kaleidoscopic coloring. It is shown that for each integer $k \geq 3$, the complete graph K_{k+3} is a k-kaleidoscope and the complete graph K_n is a 3-kaleidoscope for each integer $n \geq 6$. The largest order of an r-regular 3-kaleidoscope is $\binom{r-1}{2}$. It is shown that for each integer $r \geq 5$ such that $r \not\equiv 3 \pmod 4$, there exists an r-regular 3-kaleidoscope of order $\binom{r-1}{2}$.

Keywords: edge coloring, vertex coloring, kaleidoscopic coloring, kaleidoscope.

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References

[1] M. Aigner, E. Triesch and Zs. Tuza, *Irregular assignments and vertex-distinguishing edge-colorings of graphs*, Combinatorics' 90 Elsevier Science Pub., New York (1992) 1–9.

[2] A C. Burris and R.H. Schelp, *Vertex-distinguishing proper edge colorings*, J. Graph Theory **26** (1997) 73–82.
doi:10.1002/(SICI)1097-0118(199710)26:2⟨73::AID-JGT2⟩3.0.CO;2-C

[3] J. Černý, M. Horňák and R. Soták, *Observability of a graph*, Math. Slovaca **46** (1996) 21–31.

[4] G. Chartrand. S. English and P. Zhang, *Binomial colorings of graphs*, Bull. Inst. Combin. Appl. **76** (2016) 69–84.

[5] G. Chartrand and P. Zhang, Chromatic Graph Theory (Chapman & Hall/CRC Press, Boca Raton, FL, 2009).

[6] O. Favaron, H. Li and R. H. Schelp, *Strong edge colorings of graphs*, Discrete Math. **159** (1996) 103–109.
doi:10.1016/0012-365X(95)00102-3

[7] P.G. Tait, *Remarks on the colouring of maps*, Proc. Royal Soc. Edinburgh **10** (1880) 501–503, 729.
doi:10.1017/S0370164600044643

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